

AD-A082 049

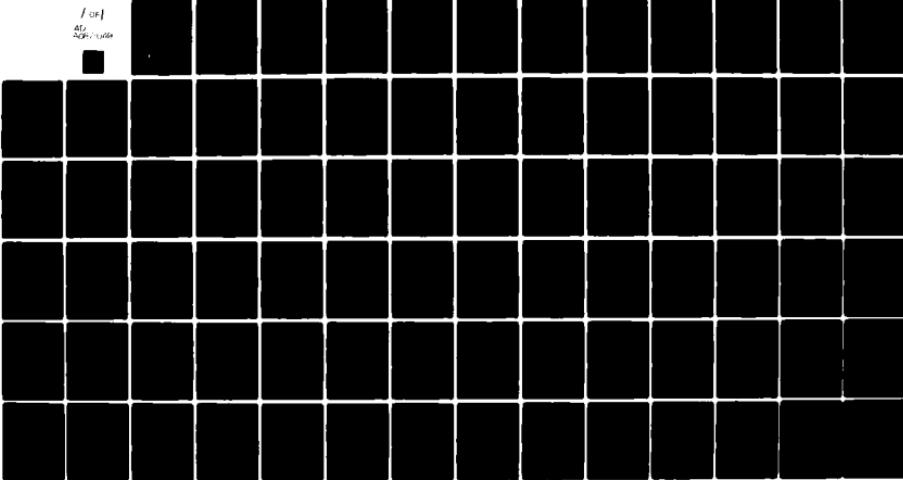
NAVAL SURFACE WEAPONS CENTER SILVER SPRING MD  
DYNAMIC BUCKLING OF CYLINDRICAL SHELLS: A LITERATURE SURVEY. (U)  
OCT 79 D B GRENIER

UNCLASSIFIED NSWC/TR-79-447

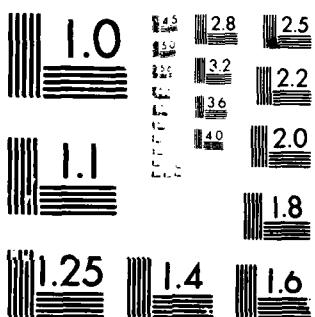
F/G 20/11

NL

1 OF 1  
NSWC/TR-79-447



END  
DATE  
PLATE  
4-80  
BYNC



MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963

ADA 082049

NSWC TR 79-447



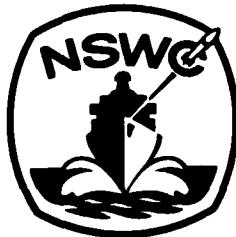
## DYNAMIC BUCKLING OF CYLINDRICAL SHELLS: A LITERATURE SURVEY

BY DAVID B. GRENIER  
UNDERWATER SYSTEMS DEPARTMENT

LEVEL 1

23 OCTOBER 1979

Approved for public release, distribution unlimited.



### NAVAL SURFACE WEAPONS CENTER

Dahlgren, Virginia 22448 • Silver Spring, Maryland 20910

80 3 - 19 022

DRAFT

## UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 14 NSWC/TR-79-447	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 Dynamic Buckling of Cylindrical Shells: A Literature Survey	5. TYPE OF REPORT & PERIOD COVERED 9 Final report	
7. AUTHOR(s) 10 David B./Grenier	6. PERFORMING ORG. REPORT NUMBER 16 SR02303	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center White Oak Silver Spring, Maryland 20910	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Element 61153N; Project Number SR02303; Task Area Number SR02303 Work Unit Number U54AB	
11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE 11 23 October 1979	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	13. NUMBER OF PAGES 12 82	
15. SECURITY CLASS. (of this Report) UNCLASSIFIED		
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release, distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Dynamic Loads Stability Survey Shells Cylindrical bodies		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A literature survey was conducted in order to ascertain the current theoretical methods available for predicting dynamic buckling behavior of cylindrical shells. Existing theories which give consideration to advanced shell wall geometries and materials are examined. Emphasis is placed on theories which lend themselves either to a closed form solution or to an iterative solution which does not require finite difference or finite element modeling. →		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 68 IS OBSOLETE  
S/N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

411503

JDC

**UNCLASSIFIED**

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. (Cont.)

The task of developing such theories appears to be so overwhelming that few investigators stepped further than the bounds of a simple monocoque shell subjected to end or side-pulse loading. A brief summary and critique of each significant article is included. Further work is directed in the modification of existing programs to create general dynamic shell design criteria.

Accession For	
NTIS Q&I	
DOC TAB	
Unclassified	
Justification _____	
By _____	
Distribution/ _____	
Availability/ _____	
Dist	Available/or special
	P

**UNCLASSIFIED**

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

FOREWORD

A literature search was performed to assess the state of the art of the effort to create design criteria dealing with dynamic buckling of cylindrical shells. This work was initiated to fill the need of the Navy to have design tools to use in making missiles, mines, and sonobuoys. After a comprehensive search, articles were critically reviewed and evaluated on their ability to produce design criteria to meet the Navy's requirements. Several recent papers indicated that the necessary theories were available but had not been applied to producing the parametric studies needed.

The search included all available computer software. Several programs were found to contain the theories and ability to generate the desired design information. Modifications are required, however, to make the programs useful to the present study.

  
GEORGE P. KALAF  
By direction

CONTENTS

	<u>Page</u>
Introduction.....	7
Historical Review.....	8
- Static Buckling.....	8
- Dynamic Buckling.....	10
Literature Evaluation.....	17
Recommendations For Further Efforts.....	20
Appendix A Summary of Literature Sources Searched.....	A-1
Appendix B Literature Survey Notes.....	B-1
Appendix C Results of COSMIC Computer Program Search.....	C-1
Appendix D Initial Imperfections Summary.....	D-1

## INTRODUCTION

Numerous ordnance items, such as missiles, mines, and sonobuoys, are subjected to axial impact loads resulting from water entry or through-ice penetration. The use of modern lightweight, high strength materials for these applications tends to decrease shell stability unless adequate stiffening is employed. Accurate prediction of the transient behavior of shells subjected to axial and side pulse loads therefore becomes of paramount importance if structural weight is to be minimized.

At present there are a number of computer codes (STAGS, for example) which handle the dynamic buckling problem by discretizing the structure through the use of finite differences or elements. These codes can hardly be considered design tools, because the user must have some idea of a design prior to establishing a model. In addition, these computer codes are expensive to use and provide no means for cost-effectively conducting any type of parametric design study; that is, if a given design proves to be over- or under-designed, there is no clear indication of the options available to the designer in order to weight-efficiently redesign the structure.

A literature survey was therefore conducted to ascertain the current theoretical methods available for predicting dynamic buckling behavior. It is presented in three parts: 1) a historical review, giving, in brief, a look at the state-of-the-art, 2) an evaluation of the state-of-the-art with a critical look at specific problem areas and 3) recommendations for further activities in the field.

## HISTORICAL REVIEW

The subject of dynamic buckling of cylindrical shells has interested many researchers for varying reasons. In the past twenty years the aero-space industry has shown the greatest interest because of the economic advantages associated with improved designs. The defense industry has supported efforts commensurate with its needs in missile and ordnance technology. Others in the scientific community have received aid from various organizations enabling them to advance the state-of-the-art. Whatever the motives, the results include papers which form the foundation for further efforts.

The work compiled herein can be grouped into general categories in several ways. One method is by the type of loading the shell is subjected to, static or dynamic. Other ways include division by type of solution or by type of shell. When organized by type of loading and put in chronological order, one can trace the general philosophies used in dealing with the phenomenon. Any survey of dynamic buckling literature must contain literature of the static case because many authors either simply extend static theory to include the dynamic problem, or use insight gained from the static buckling phenomenon to begin their study of dynamic buckling.

The following historical synopsis is organized by loading criteria and covers literature published since 1960. The year 1960 has been selected because of the significant efforts begun in the 60's which have become the basis for present day programs. The papers included constitute the most significant programs of the period. A summary of literature sources is given in Appendix A. Notes taken on each article highlighting the important aspects of the performed research are presented in Appendix B.

STATIC BUCKLING

Early researchers such as Donnell, von Karman, Tsien, and Koiter had developed theories, both linear and non-linear, concerning the buckling of circular cylindrical shells. These theories have their basis in continuum mechanics and stability. Their strength lies in their general form and versatility, allowing the inclusion of varying shell geometries and loading sequences.

One of the first papers to be published in the field of static buckling of shells in the 1960's was written by Brush and Morton in 1961.<sup>1</sup> The work was based on experiment and treated the shell as a column with an eccentric load. The results were not important to this study but the simplifying assumptions made give a good indication of the abilities of the researchers to attack the problem in a realistic manner at that time.

---

<sup>1</sup>Brush, D. O. and Morton, F. G., "Stress & Stability Analysis of Cylindrical and Conical Shells," Tech. Report LMSD-894808, Lockheed Missiles and Space Div., Sunnyvale, CA, 31 May 1961.

In attempting to analyze the complex combined forces of axial compression and external pressure encountered in launch vehicle structures, Lakshmikantham and Gerard, in 1964, wrote an analysis of unstiffened circular cylindrical shells based upon the Donnell equation in terms of normal displacements.<sup>2</sup> The analysis included initial imperfections and viewed the buckling stress in terms of the Batdorf parameter, Z. Imperfections or prebuckling deformations were noted to have an unusual influence on the structures, particularly when an axial load was combined with at least a small circumferential load, i.e., axial loads made the shell imperfection sensitive while the shell was relatively insensitive to imperfections under a pure circumferential load.

The method of Singer, Baruch, and Harari which would be used by others later, dealt with shell ring and stringer stiffeners.<sup>3</sup> It was the first to "smear" the equivalent stiffening effect of the stiffener over the entire shell. It is interesting that for purposes of analysis it was assumed that stiffeners were closely spaced so that the smearing took care of both inside and outside effects, even though the author concluded that there were considerable differences between inside and outside stiffeners. Small deflection theory using Donnell type stability equations was utilized.

In 1967 the direct method of Liapunov using Donnell-type shell theory was used to analyze a specific shell by making many simplifying assumptions.<sup>4</sup> But it was Hutchinson and Amazigo<sup>5</sup> who made advances using non-linear Donnell-type strain displacement equations analyzing imperfection sensitivity with respect to Z for stiffened shells. The solution could not be applied to design a shell but, again, much insight was gained. One result of their investigation was the prediction of a particular imperfection sensitivity in the low Z range.

The next significant paper was written by Horton and Tenerelli.<sup>6</sup> They presented both analytical and experimental work dealing with imperfection sensitivity under axial and compressive radial loads. Experimentally they showed that

<sup>2</sup>Lakshmikantham, C. and Gerard, G., "Elastic Stability of Cylindrical Shells Under Axial and Lateral Loads," Journal of Royal Aerospace Society Vol. 68, Nov 1964, pp. 773-775.

<sup>3</sup>Singer, J., Baruch, M. and Harari, O., "On the Stability of Eccentrically Stiffened Cylindrical Shells Under Axial Compression," International Journal of Solids and Structures, Vol. 3, Oct 1967, pp. 445-470.

<sup>4</sup>Hegemier, G. A., "Stability of Cylindrical Shells Under Moving Loads by the Direct Method of Liapunov," Office of Naval Research, Contract N00014-67-A-0109-0003, Task NR 064-496 Tech. Report No. 5, Dept. of Aerospace and Mech. Eng. Sci., Univ. Cal., San Diego, Jan 1967.

<sup>5</sup>Hutchinson, J. W. and Amazigo, J. C., "Imperfection Sensitivity of Eccentrically Stiffened Cylindrical Shells," AIAA Journal, Vol. 5, March 1967, pp. 392-401.

<sup>6</sup>Horton, W. H. and Tenerelli, O. J., "The Instability of Unstiffened and Ring Stabilized Thin Walled Circular Cylinders Under Non-Uniform Axial Load Conditions," Israel Journal of Technology, Vol. 6, No. 1-2, 1968, pp. 11-24.

the critical compressive stress was 30 to 40% less than the critical bending stress and that due to imperfections, an increased allowable stress could be used in flexure if it were possible to assess the probability that the location of the weakest section of the shell would not correspond to the region of maximum stress. The analysis was accomplished using a computer program which solved the three simultaneous equations of Flügge.

M. A. Shumik published a paper<sup>7</sup> claiming to attack the dynamic stability of cylindrical shells under the action of dynamic radial pressure. He assumed inertia could be neglected thereby making it a quasi-static problem. This depicts a common problem associated with buckling analysis, that is, the results of assumptions. El Naschie<sup>8</sup> encountered this same hurdle in his mathematical treatment of buckling patterns using a two dimensional Riemann space model.

The current state of the art in static buckling analysis is presented by Weller and Singer.<sup>9</sup> By comparing experimental and theoretical results it is shown that linear theory can be used effectively as a first approximation for predicting buckling loads of stringer stiffened shells. For the analysis, general instability was calculated using the concept of "smeared" stiffeners. The results of the experimentation refuted the prediction by Hutchinson of an imperfection sensitivity in the low Z range.

#### DYNAMIC BUCKLING

One of the first American papers to appear in the field of buckling of cylindrical shells under dynamic loads was written by J. C. Yao in 1961.<sup>10</sup> This work was followed by another in 1963.<sup>11</sup> Both were analytical using Mathieu's equation and a modified Donnell equation of motion to examine the stability of the system under certain assumptions by seeking two wave parameters. For his first paper the flexural mode was coupled with the axisymmetric mode and was shown to respond in an oscillatory or exponential fashion depending upon the magnitude of the square wave pressure pulse. In the second analysis periodic load frequencies were assumed to be very much less than the frequencies of the stress

<sup>7</sup>Shumik, M. A., "The Stability of Cylindrical Shells Under the Action of Dynamic Radial Pressure," Foreign Technology Division, Wright-Patterson AFB, Ohio, 25 Jun 1974, 13 pp.

<sup>8</sup>El Naschie, M. S., "Localized Diamond-Shaped Buckling Patterns of Axially Compressed Cylindrical Shells," AIAA Journal, 13, 6, pp. 837-838, (Technical Notes), Jun 1975.

<sup>9</sup>Weller, T. and Singer, J., "Experimental Studies on the Buckling Under Axial Compression of Integrally Stringer Stiffened Circular Cylindrical Shells," Journal of Applied Mechanics, 44, Dec 1977, pp. 721-730.

<sup>10</sup>Yao, John C., "Stability of a Cylinder Under Dynamic Radial Pressure," A.R.S. Journal, Vol. 31, No. 12, Dec 1961, pp. 1705-1708.

<sup>11</sup>Yao, John C., "Dynamic Stability of Cylindrical Shells Under Static and Periodic Axial and Radial Loads," AIAA Journal, Vol. 1, No. 6, Jun 1963, pp. 1391-1396.

waves in the cylinder and the cylinders were of medium length, ( $10^2 < Z < 10^4$ ). Regions of stability were examined for cylinders with no stiffeners where the form of the displacement was assumed. End conditions were ignored at this point whereas by 1977 they would be considered important parameters.<sup>9</sup>

The question of whether to consider the effects of inertia in the dynamic problem was generally answered by making assumptions. Roth and Klosner in 1964<sup>12</sup> assumed the effects of inertia negligible in their study of the nonlinear response of cylindrical shells subjected to axial loads of constant magnitudes of prolonged or finite time duration. The analysis used Donnell's strain-displacement relationships, Hooke's law for thin shells, equations for elastic, extensional strain energy, Hamilton's principle, and an Airy stress function to give two nonlinear differential equations in terms of the stress function and radial displacement. This method resulted in the first data related to impulse loading. The critical impulse was found to decrease as the time duration of loading decreased and the dynamic buckling stress decreased with an increase in the time duration of the loading. The lack of inertia terms, however, leave some room for doubt concerning the accuracy of the analysis for all loads and shells.

The first significant paper concerning cylindrical shells under axial impact which includes experimentation was written by Linberg and Herbert in 1966.<sup>13</sup> Linear theory was applied to allow the buckling form to assume its own shape. Simplified Donnell-type equations were used which incorporated imperfections. The experimentation was run simultaneously with the theoretical investigation to see if the shell, once deformed into its initial deflection pattern defined by linear shell theory, would continue in that form. The experimentation used a unique method of applying explosives to accelerate the shell while a high speed camera monitored its motion at 240000 frames per second. The loading was uniform and controlled to within 2 microseconds. The experimentation confirmed the premise that early small-deflection buckling dictates the pattern into which large deflection buckles form.

Further experimentation was performed by Coppa<sup>14</sup> using cylinders set on a stand pressurized with a free end and impacted by a drop test apparatus. The test cylinders were unstiffened and assumed perfect. Like the article by Roth and Klosner<sup>12</sup> the initial mode of buckling was examined and certain insights were

<sup>9</sup> See footnote 9 on page 10.

<sup>12</sup> Roth, R. S. and Klosner, J. M., "Nonlinear Response of Cylindrical Shells Subjected to Dynamic Axial Load," AIAA Journal, Vol. 2, Oct 1964, pp. 1788-1794.

<sup>13</sup> Linberg, H. and Herbert, R. E., "Dynamic Buckling of Thin Cylindrical Shells Under Axial Impact," Journal of Applied Mechanics, Vol. 33, Jan 1966, pp. 105-112.

<sup>14</sup> Coppa, A. P., "Effect of End Conditions on the Buckling of Cylindrical Shells Under Axial Compressive Impact," G.E. Missile and Space Div., Space Sciences Laboratory, R66SD24, May 1966, 22 pp.

gained. It was postulated that triangular buckles represent a lower energy mode of response than the symmetric mode. Symmetrical buckles result from predominantly outward displacements, triangular buckles inward. When a shell is impacted, it is forced to respond quickly in an energetically efficient deformation mode. Therefore the dominant mode is symmetric initially. Whether this mode prevails depends upon: shell thickness, amount of lateral support, and the relationship between impacted strain and the yield strain of the material. It was also noted that triangular buckling was independent of end conditions which had only been hypothesized previously.

The first paper to include inertia, imperfections, and stiffeners, dealing with dynamic buckling used large deflection theory and assumed clamped ends.<sup>15</sup> The "smearing" technique was used to deal with the stiffeners and assumed forms were made for radial displacements and end shortening.

Under the guidance of Professor R. E. Ball, C. C. Cromer<sup>16</sup> considered the nonlinear dynamic response of ring stiffened shells. Inertia was not included but imperfections were in the nonlinear Donnell-type equations. Damping due to viscous forces was also incorporated. This resulted in a lengthy computer program for ring stiffened shells under axisymmetric radial loads only.

Another computer program using a modified membrane theory including transverse shear deformation was written by Mortimer, Rose, and Chou in 1969.<sup>17</sup> Tests were also conducted to verify the computer output using a pendulum impactor, photodiodes and light source. The analysis includes no inertia or imperfection effects and was for simple unstiffened shells.

In 1972, Professor R. E. Ball<sup>18</sup> presented a computer program using Sander's nonlinear thin shell theory for conditions of small strains and moderately small rotations. The program could analyze shells of revolution for any arbitrary load definable by a Fourier series. It approximated dependent variables using Fourier series and treated nonlinear modal coupling with pseudo loads. Limitations included its application only when geometric and material properties were axisymmetric,

---

<sup>15</sup> Dietz, W. K., "On the Dynamic Stability of Eccentrically Reinforced Circular Cylindrical Shells Technical Report, Sep 1, 1965 - Jan 31, 1967," Syracuse University Research Institute, NY, Applied Mechanics Lab., 31 Jan 67, 190 pp.

<sup>16</sup> Cromer, Charles Clinton, "An Investigation of the Nonlinear Dynamic Response of Cylindrical Shells Under Transient Pressure," Naval Post Graduate School, Monterey, CA, April 1969, 281 pp.

<sup>17</sup> Chou, P. C., Mortimer, R. W. and Rose, J. L., "Wave Propagation in Stepped and Joined Shells Annual Report, 1 Sep 1969," Drexel Institute of Technology, Philadelphia, PA, Wave Propagation Research Center, 24 Oct 1969, 37 pp.

<sup>18</sup> Ball, R. E., "A Program for the Nonlinear Static and Dynamic Analysis of Arbitrarily Loaded Shells of Revolution," Computers and Structures, Vol. 2, pp. 141-162, Pergamon Press 1972.

the material was isotropic, and shells had no stringers. The results demanded interpretation to ascertain whether the shell reached its buckling mode. Subsequent articles by Professor Ball dealt with similar subject matter but were not pertinent to this study.<sup>19,20,21</sup>

Other analytic approaches included the use of extremely general field equations<sup>22</sup> and an energy approach to examine the stability of arches and spherical caps.<sup>23</sup> In the analysis using the energy method it was mentioned that complex structures, imperfections, and real material properties pose problems to the successful use of this approach.

The 4 degree of freedom model, the nonlinear Donnell equations, and the basic method of solution performed by Roth and Klosner<sup>12</sup> in 1964 were reworked by Y. S. Tamura in 1973.<sup>24</sup> As with Roth and Klosner, stiffeners were neglected. However, unlike them, inertia was approximated, damping of the axial motion was included, and parametric studies of wave number, mass of loaded edge, and damping factor were conducted. It was concluded that due to frequency coupling between axial and radial motions, axial inertia plays an essential role in characterizing the dynamic instability of a finite length shell. This was in direct opposition to all previous assumptions about inertia effects.

<sup>19</sup>Ball, R. E., "Survivability of the Five-Inch Gun Launched Finned Motor Case," Naval Post Graduate School, Monterey, CA NPS-57Bp72081A, 10 August 1972.

<sup>20</sup>Ball, R. E. and Burt, J. A., "Dynamic Buckling of Shallow Spherical Shells," J. Applied Mechanics, June 1973, pp. 411-416.

<sup>21</sup>Ball, R. E., et al., "A Comparison of Computer Results for the Dynamic Response of the LMSC Truncated Cone," Computers and Structures, Vol. 4, pp. 485-498, Pergamon Press 1974.

<sup>22</sup>Darevskii, V. M., "Stability of a Cylindrical Shell to Dynamic Axial Loading," Mekh. Tverd. Tela, (USSR), No. 2, 1973, pp. 162-172.

<sup>23</sup>Bhatia, P. and Babcock, C. D., Jr., "Dynamic Buckling of Structures," California Institute of Technology, Pasadena Graduate Aerospace Labs. Feb 1973, 64 pp.

<sup>12</sup>See footnote 12 on page 11.

<sup>24</sup>Tamura, Y. S., "Dynamic Stability of Cylindrical Shells Under Step Loading," Air Force Office of Scientific Research, Calif. Inst. of Tech., AD-777-291, June 1973.

Approaching the subject from an elastic systems viewpoint the nonlinear von Karman-Donnell equations can lead to a general analysis of the phenomenon of resonance and its importance to system stability. One such analysis was conducted by Lange<sup>25</sup> of a specific shell structure to show that nonlinear coupling of normal modes of the solutions to the equilibrium equations can have significant bearing on the stability of elastic systems.

C. A. Fisher and C. W. Bert<sup>26</sup> developed a computer program which neglected imperfections and treated outside stiffeners perfunctorily, but included discrete rings and stiffeners subjected to rapidly applied axial compressive loads. Both general and localized instability were considered. The major drawback was that the program used a tremendous amount of computer time and core space.

Publishing at the same time as Tamura<sup>24</sup>, Tsui and Lakshmikantham had written their first in a series of articles dealing with the nonlinear and linear response of stiffened shells to dynamic loads.<sup>27</sup> Tamura had worked for the Air Force Office of Scientific Research and Tsui and Lakshmikantham for the Army Materials and Mechanics Research Center. Independently they used the same approach to solve very similar problems: Tamura, cylindrical shells; Tsui, stiffened shells. The articles by Tsui and Lakshmikantham investigated the use of the von Karman-Donnell non-linear approach<sup>27</sup>, the linear theory of Singer<sup>28</sup>, and again the Donnell type formulation.<sup>29,30</sup> In their first paper they indicated that dynamic buckling loads are less for inside stiffened shells, a result similar to that of Hutchinson and

<sup>25</sup> Lange, C. G. and Newell, A. C., "The Role of Resonant Interactions in the Dynamic Stability of Elastic Systems," California University, Los Angeles, Department of Mathematics, 1973, 13 pp.

<sup>26</sup> Fisher, C. A. and Bert, C. W., "Design of Crashworthy Aircraft Cabins Based on Dynamic Buckling," J. Aircraft, Vol. 10, No. 11, November 1973, pp. 693-695.

<sup>24</sup> See footnote 24 on page 13.

<sup>27</sup> Tsui and Lakshmikantham, "The Nonlinear Dynamic Response of Stiffened Shells Under Compressive Axial Impulse," Applied Mechanics, 4th Canadian Congress, (CANCAM 73) 2 May - 1 June 1973, Montreal, Canada.

<sup>28</sup> Tieu-Yu Tsui, et al, "Linear Stability of Eccentrically Stiffened Cylindrical Shells Under Axial Compression," Army Mat. & Mech. Research Center, Watertown, Mass., AD-775-445, Dec. 1973.

<sup>29</sup> Lakshmikantham, C. and Tsui, T., "Dynamic Stability of Axially-Stiffened Imperfect Cylindrical Shells Under Axial Step Loading," AIAA Journal, Vol. 12, Feb 1974, pp. 163-169.

<sup>30</sup> Lakshmikantham, C. and Tsui, T., "Dynamic Buckling of Ring Stiffened Cylindrical Shells," Army Materials and Mech. Research Center, Watertown, MA, AIAA Journal, Vol. 13, No. 9, Sep 1975, pp. 1165-70.

Amazigo<sup>5</sup> for the static case. It is in these papers that is found the first efforts to quantify the effects of stiffener eccentricity. Eccentricity is defined as the measure of the distance between the centroid of the stiffener and the middle surface of the shell with sign (+ or -) indicating inside or outside orientation. In all cases the computer was used to do parametric studies using tremendous amounts of computer time.

Just prior to the last of the articles by Tsui and Lakshmikantham<sup>30</sup> Tamura co-authored a paper with Babcock<sup>31</sup> dealing again with the subject of dynamic buckling of unstiffened cylindrical shells. Both treatises<sup>30,31</sup> dealt with the subject in almost identical fashions: using nonlinear Donnell's equations in terms of displacements, assuming radial displacement and initial imperfection functions, they evaluated the energy of the system, used Hamilton's principle, and then used an approximation method to integrate the resulting nonlinear ordinary differential equations. In both cases axial inertia was found to have an important influence on the final buckling characteristics of the shell. Tamura also performed a static analysis and, when comparing it to the dynamic analysis, found that for finite length shells reflections of the axial stress wave must be taken into account.

New innovations in the Tamura paper included assessment of the influence of a realistic triggering mechanism on the calculated dynamic buckling loads and the inclusion of in-plane inertia. Together with the progress in the areas of axial inertia, imperfection approximations, application of Donnell's equations, and numerical integration techniques this represents the most thorough nonlinear analysis of unstiffened cylindrical shells under a dynamic load to date.

Lakshmikantham and Tsui have performed the most comprehensive nonlinear analysis of stiffened cylindrical shells under a dynamic load to date.<sup>30</sup>

The question of critical impact velocity for a cylindrical shell was raised by A. B. Efimov.<sup>32</sup> He derived a formula which calculated the critical velocity of a cylindrical shell on longitudinal impact against an infinite mass. The theory was based upon a nonlinear von Karman-Donnell equation in terms of radial displacement. The author claimed that the formula agreed with experimentation to within 12%. No proof of experimentation was presented. Efimov's mentor, Grigolyuk,

---

<sup>5</sup>See footnote 5 on page 9.

<sup>30</sup>Lakshmikanthan, C. and Tsui, T., "Dynamic Buckling of Ring Stiffened Cylindrical Shells," Army Materials and Mech. Research Center, Watertown, MA, AIAA Journal, Vol. 13, No. 9, Sep 1975, pp. 1165-70.

<sup>31</sup>Tamura, Y. S. and Babcock, C. D., "Dynamic Stability of Cylindrical Shells Under Step Loading," Transactions of the ASME, March 1975, pp. 190-194.

<sup>32</sup>Efimov, A. B., Malyi, V. I. and Uteshev, S. A., "Loss of Stability of a Cylindrical Shell on Longitudinal Impact," Foreign Technology Division, Wright-Patterson AFB, Ohio, 27 Jun 1974, 12 pp.

published a paper in 1975 which was written in the same style, not displaying any evidence of real work, but investigating the effect of external pressure impulse shape on circular cylindrical shells.<sup>33</sup>

Three studies followed using linear buckling theory.<sup>34,35,36</sup> The first in 1974 considered arbitrary axisymmetric shells subject to axisymmetric periodic loads of long duration, thereby creating a pseudo-dynamic loading sequence.<sup>34</sup> The second in 1975 was filled with assumptions to disregard higher order terms to lead to Mathieu's equation in order to generate parametric curves showing minimum buckling loads.<sup>35</sup> The last, by Maymon and Libai<sup>36</sup> in 1977, linearized the Donnell-type equations derived by Baruch and Singer to yield a computer program to analyze a shell under axial impact. Similar to others who have dealt with stiffened shells, a smearing technique was used. The results included a parametric study of different shells and an analysis of directions to take in further work.

The paper of Maymon and Libai<sup>36</sup> was quickly followed by that of Libai and Durban.<sup>37</sup> Instead of following his own recommendations, Libai's second paper concerned itself with circumferentially varying axial edge loads or thermal loads as opposed to axial impact. The problem was also approached differently than before. It was treated as an eigenvalue problem derived from the linearized Donnell equations and membrane theory. If Libai had only continued his efforts along the lines of axial impact perhaps design criteria for shells subject to this loading could have been established.

Perhaps the most comprehensive treatise of all those found in the literature thus far is that of D. G. Zimcik.<sup>38</sup> His paper reports the results of experimental testing of thin walled circular cylindrical shells subjected to dynamic transient,

<sup>33</sup> Grigolyuk, E. I. and Srebovski, A. I., "Effect of Shape of External Pressure Impulse and Design Parameters of Thin Circular Cylindrical Shells on Their Stability," NISC, Washington, DC, Translation Division, Dec 1975, 8 pp.

<sup>34</sup> Kalnins, A., "Dynamic Buckling of Axisymmetric Shells," Trans. ASME Ser. E., Vol. 4, Dec 1974, pp. 1063-68.

<sup>35</sup> Goroshko, D. A. and Emel-Yanenko, V. V., "Dynamic Stability of Layered Anisotropic Shells," Prikl. Mekh. (USSR), Vol. 11, No. 7, July 1975, pp. 42-49.  
or: Sov. Appl. Mech. (USA), Vol. 11, No. 7, 720-25, Jul 1975.

<sup>36</sup> Maymon, G. and Libai, A., "Dynamics and Failure of Cylindrical Shells Subjected to Axial Impact," AIAA Journal, Vol. 15, No. 11, Nov 1977, pp. 1624-1630.

<sup>37</sup> Libai, A. and Durban, D., "Buckling of Cylindrical Shells Subject to Nonuniform Axial Loads," Journal of Applied Mechanics, 44, Dec 1977, pp. 714-720.

<sup>38</sup> Zimcik, D. G., "Stability of Circular Cylindrical Shells Under Transient Axial Impulse Loading," Institute for Aerospace Studies, Toronto University, Mar 1976, 137 pp.

axial, square wave loading of varying time duration. Special manufacturing techniques were used to make three types of shells of birefringent liquid epoxy. The types included were: geometrically near perfect, axisymmetric imperfect, and asymmetric imperfect. Shell response was used to verify predictions obtained from both linear and nonlinear analysis based upon von Karman-Donnell equations and a modified linear stress wave analysis. Experimentation and analysis are claimed to show good agreement.

In view of the fact that the objective of this study is to develop analytical methods to be used for generating a useful design tool, the literature search was expanded to include the Computer Software Management and Information Center (COSMIC).

COSMIC is operated for NASA by the University of Georgia for the purpose of making computer programs developed in the space program available to the public. The prices charged for programs by COSMIC are established in accordance with NASA policy to recover as large a portion of COSMIC's operating expenses as possible, without making programs prohibitively expensive for small firms. In actual practice, NASA subsidizes about one-third of the cost of the services provided by COSMIC.

An exhaustive survey of the COSMIC program library found several programs of interest. Abstracts of two dynamic programs and one static program which hold much promise for this study are included in Appendix C.

#### LITERATURE EVALUATION

While the preceding literature survey showed that a great deal of work has been done on the subjects of static and dynamic buckling, it can be seen that most of the work has been left open ended or completed only for the simpler of shell wall geometries. It is no surprise that this is a prevalent finding since the subject is so encompassing that it requires the resolution of complex multivariable relationships that usually takes years to complete.

Initially the major stumbling block for researchers was the creation of a mathematical model of sufficient complexity and degrees of freedom to include all of the governing parameters. The work of Roth and Klosner<sup>12</sup> hurdled this obstacle using a Donnell-type equation, Hamilton's principle, the introduction of an Airy stress function, and subsequent use of a Runge-Kutta numeric technique.

---

<sup>12</sup>See footnote 12 on page 11.

A copy of their paper is included in Appendix E. Most studies published after this one used this technique while incorporating or modifying differing assumptions.<sup>13,16,24,25,27,29,30,31,32,36,38</sup>

Inertia terms have been a source of conflict.<sup>10,11,25,29,30,31,34,38</sup> Lakshmikantham and Tsui finally admitted that "normal" inertia was important enough to include in their final paper, long after Tamura and Babcock had shown its importance.<sup>24,31</sup> Zimcik<sup>38</sup> included axial, rotary and radial inertia in his work. It is the opinion of these experts that axial inertia must be included in any dynamic buckling model.

<sup>13</sup> See footnote 13 on page 11.

<sup>16</sup> See footnote 16 on page 12.

<sup>24</sup> See footnote 24 on page 13.

<sup>25</sup> See footnote 25 on page 14.

<sup>27</sup> See footnote 27 on page 14.

<sup>29</sup> See footnote 29 on page 14.

<sup>30</sup> See footnote 30 on page 14.

<sup>31</sup> See footnote 31 on page 15.

<sup>32</sup> See footnote 32 on page 15.

<sup>36</sup> See footnote 36 on page 16.

<sup>38</sup> See footnote 38 on page 16.

<sup>10</sup> See footnote 10 on page 10.

<sup>11</sup> See footnote 11 on page 10.

<sup>34</sup> See footnote 34 on page 16.

Initial imperfections effect the buckling load and mode.<sup>2,5,6,12,13,16,23,24,27,29,30,31,32,36,38</sup> The debate concerning imperfections centers on the method of including them in the analysis, not on whether they should or should not be included. The best solutions were presented by Tamura and Babcock<sup>31</sup> who used experimental data, and Zimcik<sup>38</sup> who made and measured his own. In general the problem was dealt with by assuming some form for the imperfection. A summary of these methods is presented in Appendix D. One of the results of the Maymon-Libai paper<sup>36</sup> presented the current opinion best saying, "more effort is needed in establishing real statistical imperfection distributions." Any further effort in the field must recognize this problem and deal with it.

<sup>2</sup>See footnote 2 on page 9.

<sup>5</sup>See footnote 5 on page 9.

<sup>6</sup>See footnote 6 on page 9.

<sup>12</sup>See footnote 12 on page 11.

<sup>13</sup>See footnote 13 on page 11.

<sup>16</sup>See footnote 16 on page 12.

<sup>23</sup>See footnote 23 on page 13.

<sup>24</sup>See footnote 24 on page 13.

<sup>27</sup>See footnote 27 on page 14.

<sup>29</sup>See footnote 29 on page 14.

<sup>30</sup>See footnote 30 on page 14.

<sup>31</sup>See footnote 31 on page 15.

<sup>32</sup>See footnote 32 on page 15.

<sup>36</sup>See footnote 36 on page 16.

<sup>38</sup>See footnote 38 on page 16.

The sole use of an energy method has been ruled out except in the evaluation of instability modes.<sup>23</sup> All current work is being done using the equations of motion based on Donnell's equations, using energy methods only to satisfy boundary equations.

The results of the best papers were very similar to the objective of this study, that of the generation of parametric curves. The similarities, however, exist only in general. In all cases, the programs used to perform the studies by these authors were either very time consuming, requiring large amounts of core storage, or written in a too specific manner, not easily changed for the purposes of dealing with the objective. In no case had the parametric studies been intended for anything more than qualitative analysis.

The exhaustive survey of computer programs showed that even COSMIC does not have the type of program needed to perform the parametric study. There were a few programs which appear to lend themselves to being used as "core" programs containing the appropriate theories. These could be set up to run with a calling preprocessor to generate parametric curves. However, as yet this has not been done.

#### RECOMMENDATIONS FOR FURTHER EFFORTS

Based upon the research of previous authors presented here, further efforts in this area may be extremely fruitful. The analytical ground work, in terms of the general equations of motion and methods of solution, has been conducted, and several computer programs are available utilizing this knowledge. A further development effort could conceivably be to convert one of these programs into a design program which would vary a few of the geometric parameters, regarding weight as a merit function in the design variable space, and then to determine the minimum weight design by mathematical programming techniques.

To determine how these programs may be made useful in developing design and trade-off curves, a few of the programs and their associated documentation were purchased. Using the literature survey as background on the important aspects and acceptable theories of dynamic buckling, it was concluded that modification of one or more of these programs to yield design codes is possible. The abstracts of two dynamic programs and one static program that hold good promise for this work, SADAALSOR, SORVERT, and HOLBOAT, are included in Appendix C.

It is recommended that one or more of these programs be modified in FY80. This work is to be performed primarily by NSWC, (Mr. J. Renzi, Dr. P. Huang, and Mr. J. Matra).

An existing computer program available from NSWC Code U14 will provide the hydroelastic loading for the SADAALSOR program. It appears that the SORVERT program needs no such mating to the U14 program since it already contains hydroelastic interaction. However, the SORVERT program does not allow oblique water impact where the SADAALSOR program has no such restriction.

---

<sup>23</sup> See footnote 23 on page 13.

Before the dynamic loading case can be considered, a shell must be designed to withstand hydrostatic pressure loading. Such a design constitutes the baseline for the design procedure of the dynamic case. The purpose of including the HOLBOAT program is to have a good closed form analysis program which could become a design program, when modified, to design a weight efficient shell subject to static design loads such as hydrostatic pressure. The HOLBOAT program is the most complete and versatile program available in terms of loading and wall construction. It is also in closed form and is based upon the proper theories.

A complete research program must contain some method of verifying the accuracy of the analytical model. In this respect the NSWC facilities at White Oak have the capability and expertise to conduct a comprehensive test program of this scope. The Shock Facilities (E21) conduct water entry simulations using shock tubes. The Hydroballistics Facility (U14) performs scaled, propelled water entry tests. The explosives research division (R12) has the capability of conducting tests similar to those of Linberg and Herbert.<sup>13</sup> Full scale model tests can be done at the NOSC facility at Morris Dam, Los Angeles. The type and extent of testing must be determined by considering the data requirements resulting from the parametric study.

Some data exists which may be used in its present condition. In FY77 a water entry program was conducted in which a lightweight, high-strength case was mated to CAPTOR flight gear and air dropped twice into the Charlie Range of the Patuxent River at the NSWC Solomon's facility. The case was of sandwich construction with S-glass-epoxy laminate skins and an aluminum honeycomb core. Acceleration and strain data were gathered and film coverage was provided. This data will provide one excellent means of comparison with analytical results.

---

<sup>13</sup> See footnote 13 on page 11.

BIBLIOGRAPHY

- Ball, R. E., "A Program for the Nonlinear Static and Dynamic Analysis of Arbitrarily Loaded Shells of Revolution," Computers and Structures, Vol. 2, pp. 141-162, Pergamon Press 1972.
- Ball, R. E. and Burt, J. A., "Dynamic Buckling of Shallow Spherical Shells," J. Applied Mechanics, June 1973, pp. 411-416.
- Ball, R. E., et al., "A Comparison of Computer Results for the Dynamic Response of the LMSC Truncated Cone," Computers and Structures, Vol. 4, pp. 485-498, Pergamon Press 1974.
- Ball, R. E., "Survivability of the Five-Inch Gun Launched Finned Motor Case," Naval Post Graduate School, Monterey, CA NPS-57Bp72081A, 10 August 1972.
- Bhatia, P. and Babcock, C. D., Jr., "Dynamic Buckling of Structures," California Institute of Technology, Pasadena Graduate Aerospace Labs. Feb 1973, 64 pp.
- Brush, D. O. and Morton, F. G., "Stress & Stability Analysis of Cylindrical and Conical Shells," Tech. Report LMSD-894808, Lockheed Missiles and Space Div., Sunnyvale, CA, 31 May 1961.
- Capelli, A. P., Nishimoto, T., Pauley, K. E. and Radowski, P. P., "Study of Apollo Water Impact, Vol. 8 - Unsymmetric Shells of Revolution Analysis Final Report," North American Aviation, Inc., El Segundo, CA, Space Division, May 1967, 78 pp.
- Chou, P. C., Mortimer, R. W. and Rose, J. L., "Wave Propagation in Stepped and Joined Shells Annual Report, 1 Sep 1969," Drexel Institute of Technology, Philadelphia, PA, Wave Propagation Research Center, 24 Oct 1969, 37 pp.
- Coppa, A. P., "Effect of End Conditions on the Buckling of Cylindrical Shells Under Axial Compressive Impact," G.E. Missile and Space Div., Space Sciences Laboratory, R66SD24, May 1966, 22 pp.
- Cromer, C. C., "An Investigation of the Nonlinear Dynamic Response of Cylindrical Shells Under Transient Pressure," Naval Post Graduate School, Monterey, CA April 1969, 281 pp.
- Cromer, C. C., and Ball, R. E., "Dynamic Buckling of Cylindrical Shell," Journal of the Eng. Mechanics Division, Proceedings of the ASCE, June 1971, pp. 657-669.

BIBLIOGRAPHY (Cont.)

Darevskii, V. M., "Stability of a Cylindrical Shell to Dynamic Axial Loading," Mekh. Tverd. Tela, (USSR), No. 2, 1973, pp. 162-172.

Dietz, W. K., "On the Dynamic Stability of Eccentrically Reinforced Circular Cylindrical Shells Technical Report, Sep 1, 1965 - Jan 31, 1967," Syracuse University Research Institute, NY, Applied Mechanics Lab., 31 Jan 67, 190 pp.

Efimov, A. B., Malyi, V. I. and Uteshev, S. A., "Loss of Stability of a Cylindrical Shell on Longitudinal Impact," Foreign Technology Division, Wright-Patterson AFB, Ohio, 27 Jun 1974, 12 pp.

El Naschie, M. S., "Localized Diamond-Shaped Buckling Patterns of Axially Compressed Cylindrical Shells," AIAA Journal, 13, 6, pp. 837-838, (Technical Notes), Jun 1975.

Evensen, D. A. and Fulton, R. E., "Some Studies on the Nonlinear Dynamic Response of Shell Type Structures," NASA, Langley Research Center, Langley Station, VA, 1965, 35 pp.

Fan, T. C., "Further Study on the Dynamic Stability of Shells," Rand Corporation, Santa Monica, CA, May 1966, 2 pp.

Fisher, C. A. and Bert, C. W., "Design of Crashworthy Aircraft Cabins Based on Dynamic Buckling," J. Aircraft, Vol. 10, No. 11, November 1973, pp. 693-695.

Goroshko, D. A. and Emel-Yanenko, V. V., "Dynamic Stability of Layered Anisotropic Shells," Prikl. Mekh. (USSR), Vol. 11, No. 7, July 1975, pp. 42-49.  
or: Sov. Appl. Mech. (USA), Vol. 11, No. 7, 720-25, Jul 1975.

Grigolyuk, E. I. and Srebovski, A. I., "Effect of Shape of External Pressure Impulse and Design Parameters of Thin Circular Cylindrical Shells on Their Stability," NISC, Washington, DC, Translation Division, Dec 1975, 8 pp.

Hegemier, G. A., "Stability of Cylindrical Shells Under Moving Loads by the Direct Method of Liapunov," Office of Naval Research, Contract N00014-67-A-0109-0003, Task NR 064-496 Tech. Report No. 5, Dept. of Aerospace and Mech. Eng. Sci., Univ. Cal., San Diego, Jan 1967.

Herrmann, G., "Dynamic Stability of Structures," Northwestern University, Evanston, IL, Department of Civil Engineering, 31 Jan 1968, 322 pp.

Horton, W. H. and Tenerelli, O. J., "The Instability of Unstiffened and Ring Stabilized Thin Walled Circular Cylinders Under Non-Uniform Axial Load Conditions," Israel Journal of Technology, Vol. 6, No. 1-2, 1968, pp. 11-24.

Hutchinson, J. W. and Amazigo, J. C., "Imperfection Sensitivity of Eccentrically Stiffened Cylindrical Shells," AIAA Journal, Vol. 5, March 1967, pp. 392-401.

## BIBLIOGRAPHY (Cont.)

Kalnins, A., "Dynamic Buckling of Axisymmetric Shells," Trans. ASME Ser. E., Vol. 4, Dec 1974, pp. 1063-68.

Lakshmikantham, C. and Gerard, G., "Elastic Stability of Cylindrical Shells Under Axial and Lateral Loads," Journal of Royal Aerospace Society Vol. 68, Nov 1964, pp. 773-775.

Lakshmikanthan, C. and Tsui, T., "Dynamic Buckling of Ring Stiffened Cylindrical Shells," Army Materials and Mech. Research Center, Watertown, MA, AIAA Journal, Vol. 13, No. 9, Sep 1975, pp. 1165-70.

Lakshmikantham, C. and Tsui, T., "Dynamic Stability of Axially-Stiffened Imperfect Cylindrical Shells Under Axial Step Loading," AIAA Journal, Vol. 12, Feb 1974, pp. 163-169.

Lange, C. G. and Newell, A. C., "The Role of Resonant Interactions in the Dynamic Stability of Elastic Systems," California University, Los Angeles, Department of Mathematics, 1973, 13 pp.

Libai, A. and Durban, D., "Buckling of Cylindrical Shells Subject to Nonuniform Axial Loads," Journal of Applied Mechanics, 44, Dec 1977, pp. 714-720.

Linberg, H. and Herbert, R. E., "Dynamic Buckling of Thin Cylindrical Shells Under Axial Impact," Journal of Applied Mechanics, Vol. 33, Jan 1966, pp. 105-112.

Maymon, G. and Libai, A., "Dynamics and Failure of Cylindrical Shells Subjected to Axial Impact," AIAA Journal, Vol. 15, No. 11, Nov 1977, pp. 1624-1630.

Roth, R. S. and Klosner, J. M., "Nonlinear Response of Cylindrical Shells Subjected to Dynamic Axial Load," AIAA Journal, Vol. 2, Oct 1964, pp. 1788-1794.

Schwieger, H. and Spuida, C., "Investigations on the Dynamic Stability of Cylindrical Shells Under Axial Impact," West Germany, Jun 1070, 112 pp.

Shumik, M. A., "The Stability of Cylindrical Shells Under the Action of Dynamic Radial Pressure," Foreign Technology Division, Wright-Patterson AFB, Ohio, 25 Jun 1974, 13 pp.

Singer, J., Baruch, M. and Harari, O., "On the Stability of Eccentrically Stiffened Cylindrical Shells Under Axial Compression," International Journal of Solids and Structures, Vol. 3, Oct 1967, pp. 445-470.

Tamura, Y. S. and Babcock, C. D., "Dynamic Stability of Cylindrical Shells Under Step Loading," Transactions of the ASME, March 1975, pp. 190-194.

Tamura, Y. S., "Dynamic Stability of Cylindrical Shells Under Step Loading," Air Force Office of Scientific Research, Calif. Inst. of Tech., AD-777-291, June 1973.

BIBLIOGRAPHY (Cont.)

Tieu-Yu Tsui, et al, "Linear Stability of Eccentrically Stiffened Cylindrical Shells Under Axial Compression," Army Mat. & Mech. Research Center, Watertown, Mass., AD-775-445, Dec. 1973.

Tsui and Lakshmikantham, "The Nonlinear Dynamic Response of Stiffened Shells Under Compressive Axial Impulse," Applied Mechanics, 4th Canadian Congress, (CANCAM 73) 2 May - 1 June 1973, Montreal, Canada.

Weller, T. and Singer, J., "Experimental Studies on the Buckling Under Axial Compression of Integrally Stringer Stiffened Circular Cylindrical Shells," Journal of Applied Mechanics, 44, Dec 1977, pp. 721-730.

Yao, J. C., "Dynamic Stability of Cylindrical Shells Under Static and Periodic Axial and Radial Loads," AIAA Journal, Vol. 1, No. 6, Jun 1963, pp. 1391-1396.

Yao, J. C., "Stability of a Cylinder Under Dynamic Radial Pressure," A.R.S. Journal, Vol. 31, No. 12, Dec 1961, pp. 1705-1708.

Zimcik, D. G., "Stability of Circular Cylindrical Shells Under Transient Axial Impulse Loading," Institute for Aerospace Studies, Toronto University, Mar 1976, 137 pp.

**Appendix A**

**SUMMARY OF LITERATURE SOURCES SEARCHED**

**Computer Search by the DIALOG Service**

Database: INSPEC files 12 and 13  
Contents: The Science Abstracts  
Physics Abstracts  
Electrical & Electronics Abstracts  
Computer & Control Abstracts  
Foreign Language Source Material  
Total # of Journals scanned over 2000  
Records from 1969-1977 in file 12  
1978-Present in file 13

**Computer Search by DOD Open Literature NTIS**

Database: NTIS (file 6)  
Contents: Gov't. sponsored research, development and  
engineering plus analyses prepared by federal  
agencies:  
NASA, DDC, DOE, HEW, HUD, DOT, DOC, + 240 others.  
Anything UNCLASSIFIED.

**Library Search by D. Grenier and NSWC, WO Facilities of:**

University of Maryland Library  
White Oak Facilities  
Library of Congress

**Computer Software Management and Information Center (COSMIC)**  
**Program Abstracts**

APPENDIX B

LITERATURE SURVEY NOTES

	Page
STATIC BUCKLING	B1
DYNAMIC BUCKLING	B13
NOTATION	B45

Ref. No. 1

BRUSH, D. O. and MORTON, F. G.

May 1961

"Stress and Stability Analysis of Cylindrical and Conical Shells."

An experimental investigation of unstiffened, thin-walled cylindrical shells subjected to transverse loads applied by a cushioned saddle, this report does not even vaguely resemble the kind of information required by the present investigation.

Critique:

1. For very thin shells.
2. For surface loads induced by saddles.
3. Shell treated as a column with eccentric load
4. Results were to indicate type of saddle material required.
5. All results based on experiment.

Ref. No. 2

LAKSHMIKANTHAM, C. and GERARD, G.

March 1964

**"Elastic Stability of Cylindrical Shells Under Axial and Lateral Loads"**

Considers the forces due to a combination of axial compression and external pressure encountered in launch vehicle structures.

"Using the Donnell equation for small deformations, the report considers compressive loading combinations on the stability problem of an unstiffened circular cylinder." (p. 773)

The Donnell equation in terms of normal displacement in the absence of twisting forces:

$$D \nabla^8 w + \nabla^4 \left[ N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} \right] + \frac{Et}{R^2} \frac{\partial^4 w}{\partial x^4} = 0$$

Assumed form of w:  $w = \sin(x/\lambda_x) \sin(y/\lambda_y)$  where  $\lambda_x$  and  $\lambda_y$  are the half wavelengths of buckles along x and y directions.

Assume: for  $N_y = 0$ ,  $N_x = N$ , axial load only  $\lambda_x = L/\pi$  which means that the length of the cylinder =  $\frac{1}{2}$  a wavelength of deformation.

Let:  $\lambda_x/\lambda_y = \beta$ .

and  $\tau = N_y/N_x =$  the ratio of circumferential to axial compressive forces.

Then:

$$N_x \frac{L^2}{\pi^2} D = \frac{(1 + \beta^2)}{(1 + \beta^2 \tau)} + EtL^4 / (\pi^4 R^2 D) (1 + \beta^2 \tau) (1 + \beta^2)^2$$

Let  $Z = (L^2/Rt) (1 - v^2)^{\frac{1}{2}}$

$$k_x = 4(1 + \beta^2)^2 / (2 + 3\beta^2 \tau + \tau)$$

where  $k_x$  is the buckling stress coefficient. Then plot  $k-Z$ .

Conclusions: If small  $\tau$  values are generated as a result of initial imperfections, or prebuckling deformations, then there may be an unusual sensitivity to small values of  $\tau$ . Buckling stress is lowered as  $\tau$  increases.

Critique:

1. Assumes initial buckling pattern.
2. Unstiffened cylinders.
3. Equations for small deformations.
4. Initial imperfections dealt with only in terms of parameter  $\tau$ .

Ref. No. 3

SINGER, BARUCH, HARARI

May 1966

**"On Stability of Eccentrically Stiffened Cylindrical Shells Under Axial Compression"**

This paper studied the effect of stiffener eccentricity on the critical load for cylindrical shells under axial compression. The behavior of the eccentricity effect depends upon the geometry of the shell (height, diameter, and thickness) while the geometry of the stiffeners only influences the magnitude of this behavior.

Only 2 of the 8 possible end conditions are studied assuming that secondary boundary conditions are less pronounced in stiffened shells. Therefore only simple supports and clamped ends are examined.

Assumptions:

1. The stiffeners are distributed over whole surface.
2. Normal strains vary linearly in stiffener and sheet and are equal at their point of contact.
3. Shear membrane force carried entirely by sheet.
4. Torsional rigidity of stiffener cross section is added to that of sheet.

Donnell type stability equations for general instability initial displacements are assumed:

1. Similar to those proposed by Batdorf for clamped shell.
2. Which solve the Donnell type equation in the presence of boundary conditions.

Rings are most effective in stiffening against hydrostatic pressure. Stringers are most effective in stiffening against axial compression. Assumption: The total geometric bending stiffness of the combined stringer-shell cross-section is not affected by the position of the stringers (inside or out) even though the moment of inertia is larger for outside stringers. For closely spaced stringers this difference is very small and is neglected. Therefore, by "distributing" the stringers the author takes care of both inside and outside types because they are assumed the same.

Results: The two modes of compressive failure, symmetric and antisymmetric must be considered though the antisymmetric mode usually yields higher buckling loads except for very long and thin shells.

There are two effects of eccentricity of stringers:

1. Primary effect - outside stringers increase the extensional stiffness in the longitudinal direction more than inside stringers.
2. Secondary effect - inside stringers increase the extensional stiffness in the circumferential direction more than outside stringers.

When subjected to axial compression rings play an important role in that they act to shorten the effective length of the shell and thereby put the deformation in the short shell regime. "Now, for short shells the main resistance to buckling is in the longitudinal direction." Experimentation on 350 shells was conducted to verify these calculations.

Conclusions included:

1. Outside stringers yield higher buckling loads than inside ones for all practical geometries.
2. Eccentricity effect has a pronounced maximum at values which are common.
3. Rings alone are much less efficient as stiffeners.
4. With inside rings the buckling load is reduced by the eccentricity, whereas with outside rings, the axisymmetric pattern which is not influenced by eccentricity, dominates.
5. Behavior of the eccentricity effect depends on the geometry of the shell while the stiffeners influence its magnitude.

Critique:

1. For static case only, otherwise, much data on stiffeners giving much insight into failure modes and stiffener influence.
2. Used small deflection theory.

Ref. No. 4

HEGEMIER, G. A.

January 1967

"Stability of Cylindrical Shells Under Moving Loads By the Direct Method of Liapunov"

Investigates the stability of a long, thin, elastic, circular cylindrical shell subjected to axial compression and an axisymmetric load moving with constant velocity along the shell axis.

Governing Equations: Direct method of Liapunov using nonlinear Donnell-type shell theory.

Critique:

1. Analytic only
2. Non dynamic analyses
3. No design criteria
4. Steady state response only

Ref. No. 5

HUTCHINSON, J. W. and AMAZIGO, J. C.

March 1967

"Imperfection Sensitivity of Eccentrically Stiffened Cylindrical Shells"

This paper is a quantitative study of the effect of stringer and ring stiffening on the sensitivity of circular cylindrical shells to imperfection while subjected to axial compression and hydrostatic pressure.

Governing Equations: Nonlinear Donnell-type strain displacement.

Assumption: Imperfection is assumed in the shape of the buckling mode.

The paper goes into detailed analysis of qualitative effects of rings and stringers on imperfection sensitivity with respect to Z,  $Z = \frac{L^2}{RT}(1-v^2)^{\frac{1}{2}}$ .

Results: Under axial load, for axially stiffened shells, the imperfection sensitivity and buckling load are strongly dependent on whether stringers are inside or outside the cylinder. Stiffening can reduce or eliminate imperfection sensitivity.

Critique:

1. For static buckling case only.
2. No design information, not general.
3. Nonlinear theory.

Ref. No. 6

HORTON and TENERELLI

December 1967

"The Instability of Unstiffened and Ring Stabilized Thin Walled Circular Cylinders Under Non-Uniform Axial Load Conditions"

This work deals with the aero-space industry problems and therefore includes temperature effects. After a rather complete review of the previous works on axial and temperature induced loading of right circular cylindrical shells, tests are performed to eliminate inconsistency in the findings.

Their findings: analysis shows that critical stress should be independent of distribution whereas experimentation shows this is not true.

The critical pure compressive stress is  $\approx$  30-40% less than the critical pure bending stress experimentally.

Two experiments:

1. Beverage cans.
2. Elastic buckling using interior mandrel to restrict buckling amplitude.

Analysis was accomplished using a computer program which solved the three simultaneous equations of Flügge (displacement equations).

"The paper shows that both perfect and imperfect unstiffened thin-walled right circular cylindrical shells buckle under non-uniform load conditions when the maximum compressive stress reaches the value which would cause instability under uniform load conditions." (p. 23)

Therefore, for the uniform imperfection, the design stress level should be the same for the uniform compressed states as for the varying axial stress condition.

But due to imperfection nonuniformity, increased allowable stress may be used in flexure provided it is possible to assess the probability that the location of the weakest section of the shell will not correspond to the region of maximum stress.

Critique:

1. Unstiffened shells only.
2. Nonlinear analysis.
3. Imperfections dealt with as an aside.

Ref. No. 7

SHUMIK, M. A. (Moscow)

June 1974

"The Stability of Cylindrical Shells Under the Action of Dynamic Radial Pressure"

The study determines that there is a critical pressure impulse for a given shell.

Assumptions:

orthotropic  
ring stiffened  
disregard shear & torsional rigidity of ribs  
loading is not too "fast", therefore can disregard the tangential forces  
of inertia and the subcritical stressed state is inertialess  
initial and subsequent deflections assumed sinusoidal.

A Conclusion:

Critical pressure does not rely upon the cylinder's length.

Critique:

1. Very hard to read.
2. Radial pressure dynamic load.
3. Very slow pressure pulse is assumed so that inertia may be neglected which makes this a "quasi-static" problem, not a dynamic problem.

Ref. No. 8

EL NASCHIE, M. S.

June 1975

**"Localized Diamond Shaped Buckling Patterns of Axially Compressed Cylindrical Shells"**

This paper proposes a mathematical method of analyzing a shell based on a two-dimensional Riemann space model interpreted as the buckling of a strut on a "bending" foundation with an attenuated eigenvector. The method uses a simple differential equation with Pogorelov coefficients to result in a value of the lower critical stress ( $\sigma_c$ ) for a given buckling mode (w)

$$\sigma_c = 0.093 \frac{Et}{r(1-u^2)}$$

$$w = 2ae^{-\gamma x} \left[ (3)^{\frac{1}{2}} \cos \delta x - \sin \delta x \right]$$

where  $\gamma$  and  $\delta$  are functions of thickness (t), radius (r) and the second fundamental tensor (b):

$$\gamma = K/(2)^{\frac{1}{2}}, \quad \delta = K(\frac{3}{2})^{\frac{1}{2}}, \quad K = \frac{(30t^2\pi^3r^2)^{\frac{1}{4}}}{(b^8(1-u^2))^{\frac{1}{4}}}$$

Critique:

The schema is different but does not pursue the subject in the desired direction.

Ref. No. 9

WELLER, T. and SINGER, J.

December 1977

"Experimental Studies On the Buckling Under Axial Compression of Integrally Stringer-Stiffened Circular Cylindrical Shells"

This paper is a summary of other works by the authors.

Panels were designed using criterion derived by Koiter in 1956.

General instability calculated with "smeared" stiffener linear theory.

Conclusions:

1. Linear theory can be used effectively as a first approximation for predicting buckling loads of stringer stiffened shells.
2. Stringer stiffened shells are sensitive to boundary conditions.
3. Agreement between experiment and theory relies primarily on stiffener spacing, stiffener area ratio, and shell geometry parameter Z.
4. Material nonlinearity may have significant affect on linear theory applicability.
5. Stringer stiffened shells, within the range of geometry in practical design, are structurally more efficient than equivalent weight isotropic shells.

Critique: Static Case - eccentricity of loading avoided. Linear theory was compared with the experimentation.

Ref. No. 10

YAO, J. C.

December 1961

"Stability of a Cylinder Under Dynamic Radial Pressure"

This paper investigates the stability of an infinite cylinder under a uniform radial pressure applied as a rectangular shaped function of time.

Motion is assumed to consist of a uniform radial displacement plus one flexural mode having two waves around the circumference.

Coupling - axisymmetric mode with flexural mode.

Results: Flexural mode was shown to respond in an oscillatory or exponential manner depending on the magnitude of the pressure.

Critique:

1. End conditions assumed trivial to case examined.
2. No stiffening of any kind.
3. Not general.
4. Assumptions made to ignore inertia terms.

Ref. No. 11

YAO, J. C.

June 1963

**"Dynamic Stability of Cylindrical Shells Under Static and Periodic Axial and Radial Loads"**Assumptions:

1. The load: one static and one periodic load component only.
2. The frequency of the periodic load is << the frequency of stress waves in the cylinder.
3. Radial displacement of the form  $w = \xi(\tau) \sin n\alpha \cos k\theta$ .
4. Loading expression assumed to be  $N_1 = N_o (a \cos wt + e)$

$$N_2 = rP = rP_o (b \cos wt + f)$$

therefore defining periodic end loads and radial pressures.

5. Loads have identical frequencies.
6. Both ends are simply supported.

Using Mathieu's equation and a modified Donnell's equation of motion stability of the system with the above assumptions is examined by seeking two wave parameters.

Individual cases are examined by making other assumptions. Of the cases examined, one includes: Cylinders Under Static & Periodic Axial Loads.

Assumed: Cylinders are assumed to be of medium length so end conditions have no influence on their behavior.

Results: When  $w/w_o$  becomes larger, stability region increases.

Critique:

1. End conditions ignored, therefore not very general.
2. No stiffening of any kind.
3. Sinusoidal loading only.
4. Form of displacement assumed.

Ref. No. 12

ROTH and KLOSNER

October 1964

**"Nonlinear Response of Cylindrical Shells Subjected to Dynamic Axial Loads"**

This paper investigates the stability of circular cylindrical shells subject to uniformly distributed, suddenly applied axial loads of constant magnitude of prolonged or finite time duration. Analysis includes a 4 degree of freedom system studied numerically.

Assumptions:

1. Longitudinal inertia neglected.
2. Effect of edge restraints negligible.
3. Loading in the form of the heavyside step function.

Method: Use Donnell's strain-displacement relationships and Hooke's law for thin shells and an equation for elastic, extensional strain energy. Then the equations of motion are found by applying Hamilton's principle. Now introduce an Airy stress function. Write equilibrium and compatibility equations to give 2 nonlinear differential equations to be solved for stress function F and radial displacement w.

To solve these:

- approximate radial deflection function

$$w = h \left[ a_1(t) \cos \frac{\pi x}{\lambda} \cos \frac{\pi y}{\lambda} + a_2(t) \cos \frac{\pi x}{\lambda} + a_3(t) \cos \frac{\pi y}{\lambda} + a_4(t) \right]$$

- initial imperfections form

$$\bar{w} = h(d_1 \cos \frac{\pi x}{\lambda} \cos \frac{\pi y}{\lambda} + d_2 \cos \frac{2\pi x}{\lambda})$$

Then substitute into equilibrium and compatibility equation and apply Hamiltons principle. This yields 4 simultaneous nonlinear differential equations.

These are solved with 8 initial conditions for 5 parameters using a Runge-Kutta formula.

Results: A series of plots showing solutions to the radial displacement function versus time for given load conditions. For zero imperfections the dynamic buckling stress coincided with that of the classical linear theory. The most important factors influencing peak resistance are the initial imperfections, initial stresses, and deviations from uniform, isotropic-elastic behavior. When

a small initial displacement or a small initial velocity was assumed, a sudden jump in the amplitude of the  $a_1$  mode did not occur until the stress reached a value equal to the classical buckling stress. Initial imperfections play a dominant role in the reduction of peak stresses achieved prior to buckling. The dynamic buckling stress also decreases with increase in time duration of loading. The critical impulse  $\bar{I}_B = \bar{\sigma}_B \bar{T}$  decreases as the time duration of loading decreases.

Conclusions: Many and varied, too many to mention here.

Critique:

1. All inertia terms neglected, therefore raising doubts as to accuracy of model and applicability to our problem.
2. By assuming the modes of deflection and imperfection the model is not as general as may be desired.
3. The model was for right circular cylinders and therefore not of the general nature most desirable.
4. It did however help define governing parameters and give an idea of the directions to pursue.

Ref. No. 13

LINDBERG, H. E. and HERBERT, R. E.

March 1966

"Dynamic Buckling of a Thin Cylindrical Shell Under Axial Impact"

This paper examines buckling of thin cylindrical shells from axial impact assuming:

1. Initial imperfections can be approximated by "white noise".
2. Linear small deflection theory applies.
3. Simplified Donnell-type equations give satisfactory accuracy.
4. Shell is simply supported.
5. Imperfections and final buckled form are stationary (in space).

Theory is used to calculate the resulting growth of normal displacement modes. By using linear theory the buckling form is allowed to assume its "own" shape while deflections are small. A statistical analysis gives the expected values for the "preferred" axial and circumferential wavelengths.

Experimentation: Run simultaneously with theoretical investigation to see if the shell, once deformed into its linear shell theory initial deflection, would continue in that form. Methods included:

1. Shell free at end opposite impact.
2. Impacted end bonded to massive inside ring to provide a clamped boundary to the shell.
3. End ring accelerated explosively so that time and simultaneity of impact could be controlled to within  $\approx 2$  microseconds.
4. High speed cameras used 240,000 frames/sec.

Conclusions: Experimentation confirmed the premise that early small-deflection buckling dictates the pattern into which large-deflection buckles form.

For calculating the threshold of buckling from axial impact, it may be sufficient merely to define some magnitude of the linear-amplification function as indicative of buckling.

Critique:

1. Uniform axial load is not the same as our problem in general, but close.
2. Linear theory.
3. Initial imperfections included.
4. Simply supported shell, therefore assuming only one kind of end condition.

Later paper shows that for high speed impact ( $\text{impact} > V_c$ ) the buckling was axisymmetric.

Ref. No. 14

COPPA, A. P.

July 1966

"Effect of End Conditions on the Buckling of Cylindrical Shells Under Axial Compression Impact"

This is a report on experimentally observed effects of edge conditions on the buckling behavior of cylindrical shells under axial impact.

Experiments were performed using cylinders set on a stand, pressurized with a free end and impacted by a drop test apparatus.

Tests include different pressures and end restraints.

Results:

1. Increasing pressure increases buckling stress (to .799....).
2. Lateral restraint increases buckling stress (to .860 classical static).
3. Without restraint against inward displacements, local collapse at the impacted end occurs more rapidly.
4. Asymmetric impact precludes initial buckling response characteristic of symmetrical impact. Only small stress waves were transmitted to midlength positions and local buckling occurred at the position first struck by the impact head.
5. Buckling stress increases with velocity.

Insights: Triangular buckles represent a lower energy mode of response than the symmetrical mode. Symmetrical buckles result from predominantly outward displacements, (triangular buckles inward). When a shell is impacted, it is forced to respond quickly in an energetically economic deformation mode. Therefore the dominant mode is axisymmetric initially. Whether this mode prevails depends upon: shell thickness, amount of internal lateral support, relationship between impacted strain and the yield strain of the material.

Conclusion: The triangular response mode is the mode preferred away from the end restraint, and is independent of edge conditions.

Critique:

1. No mathematical analysis (theory).
2. No general quantitative design criteria.
3. Qualitative only.

Ref. No. 15

DIETZ, W. K.

January 1967

"On The Dynamic Stability of Eccentrically Reinforced Circular Cylindrical Shells"

An analytical investigation using large deflection theory including initial imperfections and rotatory inertia dealing with dynamic buckling loads of a clamped reinforced circular cylindrical shell.

Assumptions:

1. Stringer and ring stiffeners are closely spaced so that an "equivalent shell" approach may be used where the reinforcement effects are smeared out over the whole surface.
2. Radial displacement is assumed to be diamond patterned and satisfy clamped boundary conditions.
3. End shortening of the form  $V = V_0 e^{-\gamma t}$ .

Conclusions:

1. Rotatory inertia can be neglected in determining the critical dynamic buckling load.
2. The size of the imperfection amplitude affects the critical dynamic buckling load drastically. Whether reinforcement decreases this sensitivity or not is not addressed.
3. The effect of imperfection direction is unimportant.
4. The exponentially decaying end shortening reduces dynamic buckling load which is dependent on the time constant  $1/\gamma$ .

Critique:

1. Done for one particular shell, not general design data.
2. Dynamic case.
3. Large Deflection Theory.

Ref. No. 16

CROMER, C. C.

April 1969

"An Investigation of the Nonlinear Dynamic Response of Cylindrical Shells Under Transient Pressure"

A ring stiffened nearly circular cylindrical shell of finite length under transient, axisymmetric radial loads of arbitrary axial distribution.

Governing Equation: Nonlinear Donnell-type solved using Fourier series expansions of the dependent variables. Newmark's beta-method combined with Gauss elimination for time integration.

This paper was written for R. E. Ball, to model the Dahlgren shock tube under operational loading conditions.

Critique:

1. Ring stiffened only.
2. Axisymmetric radial loads only.
3. Inertia not considered.
4. Initial imperfections symmetric about a plane thru the axis are accounted for.
5. Nonlinear coupling between modes retained.
6. Boundary conditions consistently observed.
7. Effects of discrete eccentric rings included.
8. Viscous damping included.

Ref. No. 17

MORTIMER, R. W., CHOU, P. C. and ROSE, J. L.

October 1969

Annual Report for Wave Propagation In Stepped and Joined Shells

A computer program (MCDIT-21) was utilized to analyze shell impact problems. Impacts were characterized as longitudinal velocity step functions.

Tests were conducted using pendulum impactor, photodiodes and light source. They verified the program's output.

A modified membrane theory is used. It differs from the classical theory in that transverse shear deformation is included.

Critique:

1. No inertia effects.
2. Simple unstiffened shells.
3. No imperfection effects included.

Ref. No. 22

DAREVSKII, V. M.

1973

**"Stability of A Cylindrical Shell to Dynamic Axial Loading"**

This article examines the loss of stability by a cylindrical shell for several types of dynamic loading. The case of instantaneous axial compression is one of these.

Instability is defined as the displacements of points in the elastic body with no change in their velocities or accelerations. This occurs at a time  $t = t_0$ .

The basic equations used are the nonlinear equations of motion of the elastic body with allowance for initial deformation:

$$A_i(\sigma_1, \sigma_2, \dots) + P_i(\sigma_1, \sigma_2, \dots; u, v, w) + \pi_i(q_1, q_2, q_3; u, v, w)$$

$$+ q_i - \delta \frac{\partial^2 U_i}{\partial t^2} = 0 \quad (i = 1, 2, 3)$$

where  $\delta$  = material density

$t$  = time

$U_1 = u, U_2 = v, U_3 = w$

$q_i$  = loads

One case investigated: medium length cylindrical shell  
one edge clamped, the other loaded  
the load is evenly applied over the entire end.

The case of instantaneous axial compression is examined using a shell of "medium" length, with one edge clamped and the other under uniform axial compressive force  $-P$ , instantaneously applied.

The critical load  $P_c$  is determined using simplified secondary displacement equations analogous to the stability equations for static forces.

The aim is to find the relative value of  $P_c$  to the corresponding critical static load (in its classical sense). After much analysis:

$$\frac{1}{2} P_c^s \frac{1}{1+k} \leq P_c < \frac{1}{2} P_c^s \min(1, \frac{1}{k})$$

where:

$$k = \frac{3v(1-v^2)^{\frac{1}{4}}(1+\cos 2p)}{2\sqrt{2}\pi} \frac{1}{R} \sqrt{\frac{R}{h}},$$

No. 18

BALL, R. E.

1972

"A Program for the Nonlinear Static and Dynamic Analysis of Arbitrarily Loaded Shells of Revolution"

This paper discusses a digital computer program which analyzes the response of arbitrarily loaded shells of revolution.

Governing Equation: Sander's non-linear thin shell theory for conditions of small strains and moderately small rotations.

Dependent variables are approximated using Fourier series. Nonlinear modal coupling is treated with pseudo loads. Derivatives approximated using central finite differences. Displacement accelerations by implicit Houbolt backward difference.

Program used ONLY when:

1. Geometries and material properties are axisymmetric.
2. Applied pressure and temperature distributions can be expressed in a Fourier sine or cosine series.
3. Isotropic material,  $\nu = \text{constant}$ ,  $E$  may vary through thickness.
4. Boundaries may be closed, free, fixed or restrained.

Critique:

1. Cannot treat stringers or discrete rings.
2. Demands interpretation of results to ascertain whether shell has reached its buckling load.
3. Does not include initial imperfections.
4. Nonlinear theory.
5. Versatile boundary conditions.

R and h are the radius and thickness of the shell, and E and v are the elastic modulus and Poisson ratio, respectively.

Critique:

1. For "short" shells with thickness small so they can be classified as "medium" length.
2. No experimentation.
3. No stiffness accounted for.
4. No imperfection.
5. Boundary conditions satisfied.
6. Nonlinear theory.

Ref. No. 23

BHATIA, P. and BABCOCK, C. D. Jr.

February 1973

"Dynamic Buckling of Structures"

This report uses an energy approach to examine the stability of arches and spherical caps subject to dynamic loads.

Definition: The Dynamic Buckling Load is defined as the scalar measure of that loading history at which there is a finite jump in an appropriate measure of the response for infinitesimal increase in the load measure.

Critique: Only two degree of freedom systems are examined though it is mentioned that the theory can be applied to higher degree systems. There is one mention of an applicability problem, the theory must not apply to all structures. Also, complex structures, imperfections, and real material properties pose problems to successful use of this approach.

Ref. No. 24

TAMURA, Y. S.

June 1973

**"Dynamic Stability of Cylindrical Shells Under Step Loading"**

This paper studies dynamic stability of imperfect circular cylindrical shells under axial step loading. The study includes effects of wave number of radial mode shape, mass on the loaded edge, and damping of the axial motion.

- Axial inertia approximated
- Radial inertia included
- Radial displacement approximated by a finite degree of freedom system
- Initial imperfections given by experimental results
- Structural damping
- Critical loads determined by numerical integration of eq. of motion
- Nonlinear Donnell's equations.

Method:

1. Assume displacement function  $w$  and imperfection function  $\bar{w}$ .
2. Determine inplane displacements from their equation.
3. Evaluate total Potential & Kinetic energy.
4. Apply Hamilton's principle.

Results: Gives proper imperfection sensitivity a 4 degree of freedom model.

Conclusions: Through parametric studies of the wave number, mass, and damping factor, due to frequency coupling between axial and radial motions, the axial inertia plays an essential role in characterizing the dynamic instability of a finite length shell.

Damping suppresses parametric resonance between radial and axial modes and can bring the buckling load close to the static value.

Critique:

1. No stringer, ring inclusion.
2. Inertia terms included (axial and radial).
3. Damping included.
4. Nonlinear theory.
5. Initial imperfections included.

Ref. No. 25

LANGE, et. al.

1973

**"The Role of Resonant Interactions in Dynamic Stability of Elastic Systems"**

This is a mathematical treatise dealing with the resonance phenomenon in elastic systems. The dynamic stability of a cylindrical shell under axial compression illustrates the work. The load can be step or impulsive.

Basic Equations: Nonlinear von Karman-Donnell

The analysis includes radial inertia force. Work is done directly with the equations for radial displacement  $w$  and the Airy stress function  $f$  representing deviations from the membrane prebuckling solution for the perfect circular cylindrical shell. Time is made nondimensional:

$$t = \frac{t^*}{\sqrt{\rho R^2/E}} = \frac{t^*}{R} \sqrt{\frac{E}{\rho}}$$

(same as Tamura and Bobcock, March 1975).

where  $t^*$  is real time.

Results: Nonlinear coupling of normal modes of the solutions to the equilibrium equations can have significant bearing on the stability of elastic systems. In general the analysis investigates the phenomenon of resonance and its importance to system stability. It does not delve any further into the subject of shells than already stated.

Critique:

1. No stiffeners.
2. No imperfection analysis.
3. Theoretical only, no comparison with experimentation
4. Nonlinear theory.

Ref. No. 26

FISHER, C. A. and BERT, C. W.

November 1973

"Design of Crashworthy Aircraft Cabins Based on Dynamic Buckling"

Also: Fisher, C. A. and Bert, C. W., "Dynamic Buckling of an Axially Compressed Shell with Discrete Rings and Stiffeners", Journal of Applied Mechanics, 1973.

The authors investigate the structural integrity of aircraft structures using nonlinear analysis developed to treat dynamic buckling of a circular cylindrical shell having discrete rings and stringers subjected to rapidly applied axial compressive load. Both general or localized instability are considered.

Critique:

1. Program uses tremendous amount of computer time and core space.
2. Outside stiffeners treated perfunctorily.
3. Imperfections neglected.

Ref. No. 27

TSUI and LAKSHMIKANTHAM

June 1973

"The Nonlinear Dynamic Response of Stiffened Shells Under Compressive Axial Impulse"

The theoretical model includes:

- imperfections
- eccentricity effects

Equations: Coupled set of 4th order nonlinear von Karman-Donnell type. Integrated using 4th order Runge-Kutta scheme.

Assumptions:

1. 5-parameter modal representation of buckling pattern.
2. Asymmetric initial imperfections.

Results:

1. Shows that heavily stiffened shells are not insensitive to imperfections in the dynamic loading case.
2. Dynamic buckling loads are less with inside stiffeners (similar to static studies) such as Hutchinson and Amazigo).
3. With an increase in imperfections, the effects of stiffener eccentricity may vanish, (increasing imperfection size decreases the differences made by stiffener eccentricity (+ or -) to where eccentricity effects may vanish for the axial case and decrease considerably for the general case.

Critique: Nonlinear theory - The parametric analysis results from an involved computer program which numerically integrates four nonlinear differential equations (time steps using tremendous computer time). No inertia terms. These people are on the right track.

Ref. No. 28

TSUI, et. al.

December 1973

"Linear Stability of Eccentrically Stiffened Cylindrical Shells Under Axial Compression"

This paper considers the effect of stiffener eccentricity on buckling modes rather than buckling loads. The information is significant in that the knowledge of buckling modes allows the theoretician to have a priori knowledge of the shape of the shell failure.

The mathematics deal with:

- Stringer stiffeners, inside and out
- Ring stiffeners, inside and out

Theory Used: Linear theory of Singer which accounts for coupling between bending stress resultants and middle surface stretching, and membrane stress resultants and changes in the middle surface curvature.

- Stiffeners are smeared
- Stiffeners are essentially beam like elements
- Local instabilities ruled out.

Constitutive equations given in matrix form.

Critique: This kind of thing may be useful in the assumption of a preferred buckling mode for a given shell type.

Ref. No. 29

LAKSHMIKANTHAM and TSUI

February 1974

**"Dynamic Stability of Axially Stiffened Imperfect Cylindrical Shells Under Axial Step Loading"**

Investigates the dynamic general instability of stiffened shells under a variety of impulsive loads and deals with the case of an axially-stiffened (stringer-stiffened) cylinder under axial step loading.

Donnell-type formulation valid for moderate length cylinders

$$Z = \frac{L^2}{RH} \quad 10^2 < Z < 10^4$$

Derivation is similar to that of Baruch and Singer except for inclusion of strain-displacement relationships and initial imperfections.

- Stiffeners are close enough together to be smeared out to calculate effective rigidities.

- Beamlike elements carrying no shear.

- Local instabilities are ruled out.

Nonlinear strain-displacement relationship and initial imperfections are included.

**Conclusions:** For dynamic buckling:

1. Outside stiffeners are imperfection sensitive.
2. Imperfection sensitive structures under static load will display the same property under step loads.

Equation of motion - same as for Lakshmikantham and Tsui, 1975.

**Initial Imperfections:** a four-parameter modal approximation:

$$\bar{w} = \left(\frac{H}{R}\right) (d_1 \cos(m \bar{\pi} x) \cos(ny) + d_2 \cos(2m \bar{\pi} x))$$

$$\bar{\pi} = \pi R/L$$

Critique:

1. Stiffeners are still smeared.
2. Inertia ignored.
3. Nonlinear theory.

Ref. No. 30

LAKSHMIKANTHAM and TSUI

September 1975

"Dynamic Buckling of Ring Stiffened Cylindrical Shells"

This paper is in the same vein as previous papers by the authors.

1. Donnell's nonlinear shell theory is applicable.
2. Stiffeners are close together and therefore effective rigidities are calculated by smearing.
3. Stiffeners are beam-like elements carrying no shear.
4. No local instability failures - entire shell including stiffeners is activated at buckling.

Longitudinal and tangential inertial terms are of lower order of importance compared to normal inertia.

Buckling Criteria: A jump in the peak amplitude of the deflection is associated with a critical load.

Equations of Motion: Donnell-Karman type coupled PDE derived from the Hamilton Principle followed by Galerkin-type procedure. Use a 4 parameter modal approximation, representing diamond-shaped buckles.

The paper investigates axial and lateral step loads.

Results: Ring stiffened shells are imperfection sensitive. The essential mechanism for imperfection sensitivity in the dynamic range is the so-called "column" asymmetric mode with  $m = 1$  and  $n = 1$ .

Conclusions:

1. Axially and ring stiffened cylinders are imperfection sensitive.
2. Internal ring stiffened shells are imperfection insensitive under axial step loading. The addition of small amounts of lateral pressure reverses this effect.
3. "Ring stiffened shells under pure lateral pressure are imperfection sensitive for all ranges of ring heaviness."
4. In the dynamic range the mechanism of imperfection sensitivity is the column asymmetric mode.
5. Internal rings force the shell response into wave patterns conforming to ring spacing under axial load. Under a lateral load this effect is weakened and eventually produces the column failure mode.

Critique:

1. Stiffeners smeared.
2. Longitudinal and tangential inertia deemed unimportant.
3. Nonlinear theory used.
4. Step loads are examined. This may not be realistic.

Ref. No. 31

TAMURA and BABCOCK

March 1975

"Dynamic Stability of Cylindrical Shells Under Step Loading"

Points emphasized:

1. Effect of axial inertia is approximated.
2. Static buckling behavior is studied for the discrete model used in the dynamic analysis.
3. Experimental results are used for values of the initial imperfections.

Equations: Nonlinear Donnell's equation in terms of displacement.

Procedure:

1. Assume appropriate radial displacement function  $w$  and initial imperfection function  $\bar{w}$ .
2. Find displacements  $u$  and  $v$  from in-plane equilibrium equation and in turn  $\sigma_x$ ,  $\sigma_y$ , and  $\tau_{xy}$ .
3. Evaluate K.E. & P.E. of system.
4. Apply Hamilton's principle resulting in a nonlinear ordinary differential equation (4 degrees of freedom).

Static Analysis - To insure existance of postbuckling equilibrium positions.

In Numerical Study: Parameters to be chosen are:

1. 3 wave numbers  $i$ ,  $k$ ,  $\ell$  ( $i=2k$ )
2. Initial imperfections.

Dynamic Stability:

1. Use 4 nonlinear D.E. and initial imperfections under a special step load.
2. Use a Runge-Kutta numerical integration scheme for the starting solutions.
3. Switch to a 6th order predictor-corrector method.
4. Integrate over interval to obtain desired accuracy.

Results: Dynamic buckling load parameters are:

1. Modes of radial deformation (according to wave number  $i$ ,  $k$ , and  $\ell$ ).
2. Initial imperfections.
3. Geometric parameters and material constants  $\frac{R}{h}$ ,  $\frac{R}{L}$ , and  $\nu$ .

Axial inertia results in a dynamic buckling load much lower than static and therefore must be included in the model.

"For finite length shells, reflections of the axial stress wave must be taken into account". (p. 193)

Nondimensional time defined as  $T = \frac{t}{R} \sqrt{\frac{E}{\rho}}$

Critique:

1. First to assess the influence of a realistic triggering mechanism on the calculated dynamic buckling loads.
2. First to include in-plane inertia.
3. Nonlinear theory used.
4. Solved nonlinear partial differential equations and examined stability with respect to wave numbers resulting in Mathieu type equations.
5. No stiffeners - this analysis for unstiffened circular cylindrical shells only.

Ref. No. 32

EFIMOV, A. B., et. al.

June 1974

**"Loss of Stability of a Cylindrical Shell on Longitudinal Impact"**

This brief paper describes the phenomena of stability loss of a cylindrical shell on longitudinal impact against the end by an infinite mass.

Stability loss at the initial instant of bulging is axisymmetric and is analyzed using linearized Donnell-type equations. A random function  $w_o(x, y)$  is used to describe initial shell irregularities. The equation used is:

$$\left[ \frac{D}{h} \Delta^4 + \frac{E}{R^2} \frac{\partial^4}{\partial x^4} + \rho v c \Delta^2 \frac{\partial^2}{\partial x^2} + \rho v v c \Delta^2 \frac{\partial^2}{\partial y^2} + \rho \Delta^2 \frac{\partial^2}{\partial t^2} \right] w(x, y, t) \\ = \left[ \frac{D}{h} \Delta^2 + \frac{E}{R^2} \frac{\partial^4}{\partial x^4} \right] w_o(x, y)$$

where:  $D = \frac{Eh^3}{12(1 - v^2)}$        $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

 $R$  = radius $h$  = thickness $v$  = poissons ratio

This equation is solved to find that there are two axisymmetric waves traveling down the cylinder of which one becomes unbounded as impact velocity increases beyond a critical velocity  $v_e$ .

$$v_e = \frac{c_1 h}{R \sqrt{3(1 - v^2)}}$$

$$c_1 = \sqrt{E/\rho}$$

when  $v = 0.30$  the author claims that experiments have shown this to be true to within 12%.

(No Experimentation presented by author or claimed to have been accomplished by Author).

Critique:

1. Uniform impact across end.
2. Inelastic collision.
3. Stiffeners are not accounted for.
4. Indicates a new approach to the stability of shells on longitudinal impact.

Note: Mr. Efimov was a protege of Mr. Grigolyuk, Ref. 33.

Ref. No. 33

GRIGOLYUK, E. I.

December 1975

"Effect of Shape of External Pressure Impulse and Design Parameters of Thin Circular Cylindrical Shells On Their Stability"

To ignore inertia terms, the rate of load application is assumed greater than the rate of deflection growth.

Bends are on the order of shell thickness. The shell is viewed as an elastic system with K degrees of freedom.

Illustrated is the behavior of a shell.

Critique:

1. Avoids inertia terms.
2. No stiffeners.
3. Purely analytical but no equations or programs presented in the paper.

Ref. No. 34

KALNINS, A.

December 1974

"Dynamic Buckling of Axisymmetric Shells"

The object of this report is to extend the linear dynamic buckling analysis for arbitrary axisymmetric shells when subject to axisymmetric periodic loads of long durations.

It is limited to cases where the applied load is of sufficiently long duration when compared to the longest resonant period of the shell.

Starting with the equation of motion of a superimposed disturbance on an initial stressed shell, a series solution for the displacement is substituted and the Galerkin method used to examine flexural modes which lead to approximations of the governing equations of the form of the Hill equation and result in the Mathieu equation.

Damping is incorporated in the analysis of the response modes of the shell by incorporating it into the Mathieu equation.

The case of the Nuclear reactor cooling pipe shroud is examined.

Critique:

1. Axisymmetric shells only.
2. Limited dynamics.
3. Damping included.
4. Linear theory.

Ref. No. 35

GOROSHKO, O. A. and EMEL'YANENKO, V. V.

July 1975

"Dynamic Stability of Layered Anisotropic Shells"

Linear theory is used to study the effects of the structure of the material on the dynamic stability of orthotropic cylindrical shells subject to periodic loads for various boundary conditions.

This is a mathematical treatise filled with assumptions to make higher order terms disappear. At different times both Love's and Donnell's theories are used resulting in:

Results: Mathieu's equation and three regional equations for a given set of parameters, a set of curves was generated showing minimum critical loads.

Critique:

1. Interesting but for wrong type of loading.
2. Otherwise, the result seems to be the type of thing we are looking for.
3. Linear theory.

Ref. No. 36

MAYMON and LIBAI

November 1977

"Dynamics and Failure of Cylindrical Shells Subjected to Axial Impact"

Purpose: 1) To present a mathematical model of the behavior of cylindrical shells under axial impact including statistical characteristics of initial imperfections. 2) To present a digital computer program parametric study of different shells.

Method: Linearized Donnell-type equations derived by Baruch and Singer;

- closely spaced stiffeners to calculate displacements are smeared over the shell surface.
- at least 2 stiffeners must be in each wavelength.
- imperfections approximated by Fourier series expansions.
- stress calculations - discreteness effects taken into account,  
membrane stresses  
bending stresses
- normal distribution of initial imperfections assumed.

Failure Criteria: It was assumed that a stress oriented design limitation would be reached before any significant nonlinear effects developed.

Results: A computer program capable of using excessive displacements as failure criteria. The complexity of nonlinear analysis of stiffened shells is so great that its routine use is not practical at the present time.

Axial imperfections are most critical for dynamic buckling than the circumferential ones.

More effort is needed in establishing real statistical imperfection distributions and experimental verification of analytical modes of dynamic collapse.

Critique:

1. Linear theory (see Failure Criteria)
2. Smeared stiffeners.
3. This is the most comprehensive paper to date.
4. Computer program very large.

Ref. No. 37

LIBAI, A, and DURBAN, D.

December 1977

"Buckling of Cylindrical Shells Subjected to Nonuniform Axial Loads"

This paper deals with linear buckling of a cylindrical shell with circumferentially varying axial edge loads or thermal loads.

Closed form expressions for the eigenvalues are obtained.

The results are checked against a numerical study. The primary concern is with heating a cylinder along a strip to initiate buckling.

Linearized Donnell equations used in conjunction with membrane theory.

Critique: Not remotely near what we are looking for, but is an example of the closed form solution to the linearized Donnell equations.

Ref. No. 38

ZIMCIK, D. G.

March 1976

"Stability of Circular Cylindrical Shells Under Transient Axial Impulsive Loading"

This paper reports the results of experimental testing of thin-walled circular cylindrical shells subjected to dynamic transient axial, square wave loading of varying time duration.

Types of shells tested:

- geometrically near perfect
- axisymmetric imperfect
- asymmetric imperfect ( $\mu \approx 0.1$ )

Special manufacturing techniques were used. Material was birefringent liquid epoxy.

Results:

1. Indicated an increase in the dynamic buckling stress above the static value for all shells subjected to transient square-pulse loading.
2. Curves were drawn showing a definite finite-time buckling impulse minimum in the time domain investigated.

Shell response observed experimentally was used to verify predictions obtained from an analytic model:

- Karman-Donnell compatibility equilibrium equations.
- Galerkin procedures used to get set of nonlinear partial D.E.
- Modified linear stress wave analysis used.

Experimentation and analysis show good agreement.

Conclusions:

1. Buckling stiffness increases for short duration of loading.
2. The simplified analytical model adequately described the shells buckling response.

"The two quantities, dynamic buckling load and pulse duration can be combined as a finite-time buckling impulse to provide a conservative dynamic buckling design criterion". (p. 25)

This was first study providing experimental data on controlled asymmetric shape imperfection effects on static or dynamic buckling.

Critique:

1. Model testing may be a cost effective way to design shells to meet dynamic buckling requirements.
2. Boundary condition effects ignored in the analysis.
3. No stiffeners.
4. Inertia included (axial, rotary, and radial).
5. Both linear and nonlinear theories used.
6. Initial imperfection study is most comprehensive and relative to date.

## TERMS

c	Rate of propagation of longitudinal waves in the shell, $(E/(\rho(1-v^2)))^{1/2}$
$d_1, d_2$	Coefficients used in imperfection approximations
D	$Eh^3/12(1-v^2)$ ; flexural rigidity
E	Modulus of elasticity
F(x,y,t)	Airy Stress Function
G	Shear modulus
h,H	Shell wall thickness
i,k,l	Wave number
$k_x$	Buckling stress coefficient
L	Shell length
$N_x, N_y, N_{xy}$	Membrane stress resultants
$P_c$	Critical axial load
P	Axial load
$P_o$	Axial load coefficients
$q_i$	Loads
Q	Shear force resultant
R,r	Radius of shell curvature
$t, T, t^*$	Shell thickness, time, or nondimensional time depending on the author
$w, w_o, w$	Initial radial shape imperfection function
u,v,w	Displacements in the axial, circumferential, and radial directions

## TERMS (Cont.)

x,y,z	Axial, circumferential, and radial coordinates respectively
v	End shortening speed
z	Batdorf Parameter, nondimensional having several different forms: $L^2/Rt$ or $L^2/Rt(1-v^2)^{1/2}$
$\nu$	Poisson's ratio
$\lambda_x$	Axial half wavelength
$\lambda_y$	Circumferential half wavelength
$\nabla$	Differential operator
$\nabla^4$	$\partial^4/\partial x^4 + 2(\partial^4/\partial x^2 \partial y^2) + \partial^4/\partial y^4$
$\sigma$	Stress
$\sigma_c$	Critical stress
$\delta$	Material density, generally

**NSWC TR 79-447**

**Appendix C**

**RESULTS OF COSMIC COMPUTER PROGRAM SEARCH**

Geometrically Nonlinear Static and Dynamic Analysis of Arbitrarily Loaded  
Shells of Revolution (SADAALSOR)  
(Naval Postgraduate School)

The design of many shell structures is influenced by the geometrically nonlinear response of the shell when subjected to static and/or dynamic loads. As a consequence, a number of investigations have been devoted to the study of the buckling phenomenon exhibited by shells. Most early works examine the behavior of the shallow spherical cap, the truncated cone, and the cylinder under axisymmetric loads. Due to the lack of information on the axisymmetric response of shells with other meridional geometries and on the response of shells subjected to asymmetric loads, a computer program for the geometrically nonlinear static and dynamic response of arbitrarily loaded shells of revolution has been developed.

The program can be used to analyze any shell of revolution for which the following conditions hold:

1. The geometric and material properties of the shell are axisymmetric, but may vary along the shell meridian.
2. The applied pressure and temperature distributions and initial conditions are symmetric about a datum meridional plane.
3. The shell material is isotropic, but the modulus of elasticity may vary through the thickness. Poisson's ratio is constant.
4. The boundaries of the shell may be closed, free, fixed, or elastically restrained.

The governing partial differential equations are based upon Sanders' nonlinear thin shell theory for the condition of small strains and moderately small rotations.

At each load or time step, an estimate of the solution is obtained by extrapolation from the solutions at the previous load or time steps. The sets of algebraic equations are repeatedly solved using Potter's form of Gaussian elimination, and the pseudo loads are recomputed, until the solution converges.

An automatic variable load incrementing routine is included in the program for the static analysis. Post-buckling behavior cannot be determined in the static analysis because of the method of solution employed.

**Dynamic Response of Shells of Revolution During Vertical Impact Into  
Water - Hydroelastic Interaction (SORVERT)  
(North American Aviation, Inc.)**

This is one of several programs developed out of a study to determine the hydro-elastic response of representation of the structure of the Apollo Command Module immediately following impact on water.

The program determines the hydroelastic response of a flexible shell of revolution during an axially symmetric impact into an incompressible fluid, and is based on the following theory.

During the impact, the deformation of the body (in particular, its rate of deformation) interacts with the fluid flow to produce oscillating pressure distributions on the wetted surface of the body. The hydroelastic response of the flexible shell is obtained by the numerical solution of the combined hydrodynamic and shell equations. The results obtained are compared numerically with those derived neglecting the interaction and applying rigid-body pressures to the same elastic shell.

HOLBOAT - General Instability Analysis of Inhomogeneous, Anisotropic,  
Stiffened Cylinders Under Combined Loads  
(Martin Marietta Corp.)

The HOLBOAT computer program provides an instability analysis of inhomogeneous, anisotropic, right circular cylinder (or segment) under combined loading. It is an efficient, closed-form solution based on the Kirchoff-Love hypothesis, general anisotropic constitutive equations, and Flugge's differential equations of equilibrium. Proper use of this program is based on the user being familiar with composites technology nomenclature and theory, shell instability theory, and FORTRAN programming. The analysis for axial loads, pressure, and torsion is based upon the formulation of Cheng and Ho, who developed a solution in terms of classical small deflection theory and Flugge's differential equation of equilibrium. Their analysis has been extended to include bending.

Three operating modes are available to the user:

1. Input of experimentally determined values of stiffness from isolated axial compression, torsion, and internal pressure test.
2. Direct input of stiffness matrices.
3. Input of cylinder geometry and wall construction.

This program can be used to analyze right circular cylinders or segments of right circular cylinders. Vertical and circumferential stiffening members may be included in the modeling. Stringers or rings may be on the inside or outside of the cylinder. Wall construction may consist of a skin alone or a skin reinforced with stiffening members. The skin may be a laminated cylinder constructed of layers of different materials having different elastic properties or orientations. Each individual layer may be isotropic or orthotropic; symmetric or balanced arrangement of layers is not required.

**Appendix D**  
**INITIAL IMPERFECTIONS SUMMARY**

**Previous methods of dealing with initial imperfections:**

1. A four-parameter modal approximation

$$\bar{\omega} = (H/R) (d_1 \cos \bar{\pi}m x \cos ny + d_2 \cos 2m\bar{\pi}x) \quad \bar{\pi} = \pi R/L \quad (29,30)$$

$$\bar{\omega} = h (d_1 \cos \frac{\pi x}{\lambda_x} \cos \frac{\pi y}{\lambda_y} + d_2 \cos \frac{2\pi x}{\lambda_x}) \quad (12)$$

2. Experimental Results - actual measurements (24,31)  
or constructed to give specific values (38)

3. A random function  $\omega_o(x,y)$  (32)

4. Approximated using Fourier Series Expansions (36)

$$\omega_o = \sum_m \sum_n a_{m,n} \sin \frac{m\pi x}{L} \cos (\frac{ny}{R} - \phi_n)$$

where  $\phi_n$  = phase angle.

5. Asymmetric initial imperfections assumed (27)
6. Initial imperfections symmetric about a plane through the axis are accounted for (16)
7. Approximated by "white noise" (13)
8. Imperfection is assumed in the shape of the buckling mode (5)
9. Assumed form of initial displacement  $\omega$ : (2)

$$\omega = \sin \left( \frac{x}{\lambda_x} \right) \sin \left( \frac{y}{\lambda_y} \right)$$

**Notes:** The numbers given in parenthesis after each method refer to references given in the body of the report.

Nomenclature used on this page is defined in Appendix B.

## DISTRIBUTION

<u>Copies</u>	<u>Copies</u>		
Gould, Inc. Ocean Systems Division Attn: Peter B. Thatcher 18901 Euclid Avenue Cleveland, OH 44117	1	Director Defense Research and Engineering Department of Defense Washington, DC 20301	1
Lockheed Palo Alto Research Laboratory Attn: David Bushnell Dept. 52-33, Bldg. 205 3251 Hanover Street Palo Alto, CA 94304	1	Commanding Officer Naval Underwater Systems Center Attn: Mr. G. Rossi Newport, RI 02840	1
McDonnell Douglas Astronautics Company Attn: K. A. Heinly J. C. Peck 5301 Bolsa Avenue Huntington Beach, CA 92647	1	Commander Naval Ocean Systems Center Attn: Code 4473-B, Bldg. 150 Mr. P. Burke, Code 932 San Diego, CA 92152	1
Assistant Secretary of the Navy (Research, Engineering, and Systems) Department of the Navy Washington, DC 20350	1	Defense Documentation Center Cameron Station Alexandria, VA 22314	12
Chief of Naval Operations Department of the Navy Attn: OP-981 OP-374 Washington, DC 20350	1	Office of Naval Research Attn: Dr. Basdekas, Code ONR 607 800 N. Quincy Street Arlington, VA 22217	1
Commander Naval Sea Systems Command Attn: NAVSEA 51C PMS 406 PMS 407 SEA 09G32 SEA 03B	1 1 1 2 1	Commander Naval Air Systems Command Attn: Mr. H. Lehman, Code 530C1 LCDR D. Hunter, Code AIR-5413 Washington, DC 20361	1 1
Department of the Navy Washington, D.C. 20362		Naval Air Test Center Attn: Mr. C. Noell, Code SY-90 Patuxent River, MD 20670	1

DISTRIBUTION (Cont.)

Copies

General Electric Company  
Re-entry and Environmental  
Systems Division  
Attn: Donald Hendershott  
3198 Chestnut Street  
Philadelphia, PA 19101

1

Honeywell Incorporated  
Defense Systems Division  
Attn: Elmer G. Johnson  
Mail Station MN11-2961  
600 2nd Street, N.E.  
Hopkins, MN 55343

1

Lockheed Georgia Co.  
Attn: Mr. D. Boyles  
Dept. 72-20  
Zone 415  
86 S. Cobb Drive  
Marietta, GA 30063

1

Gould, Inc.  
Attn: Ron Seipel  
1745 Jeff Davis Highway  
Arlington, VA 22202

1

Library of Congress  
Washington, D.C. 20540  
Attn: Gift and Exchange Division

4

TO AID IN UPDATING THE DISTRIBUTION LIST  
FOR NAVAL SURFACE WEAPONS CENTER, WHITE  
OAK LABORATORY TECHNICAL REPORTS PLEASE  
COMPLETE THE FORM BELOW:

TO ALL HOLDERS OF NSWC/TR 79-447  
by David B. Grenier, Code U-11  
DO NOT RETURN THIS FORM IF ALL INFORMATION IS CURRENT

A. FACILITY NAME AND ADDRESS (OLD) (Show Zip Code)

NEW ADDRESS (Show Zip Code)

B. ATTENTION LINE ADDRESSES:

C.

REMOVE THIS FACILITY FROM THE DISTRIBUTION LIST FOR TECHNICAL REPORTS ON THIS SUBJECT.

D.

NUMBER OF COPIES DESIRED \_\_\_\_\_

DEPARTMENT OF THE NAVY  
NAVAL SURFACE WEAPONS CENTER  
WHITE OAK, SILVER SPRING, MD. 20910

OFFICIAL BUSINESS  
PENALTY FOR PRIVATE USE, \$300

POSTAGE AND FEES PAID  
DEPARTMENT OF THE NAVY  
DOD 316



COMMANDER  
NAVAL SURFACE WEAPONS CENTER  
WHITE OAK, SILVER SPRING, MARYLAND 20910

ATTENTION: CODE U-11

